

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

(In the Name of Allah, the Most Merciful, the Most Compassionate.)

PHYSICS

11



**PUNJAB EDUCATION, CURRICULUM,
TRAINING AND ASSESSMENT AUTHORITY**

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Measurements

Learning Objectives

After studying this chapter, the students will be able to:

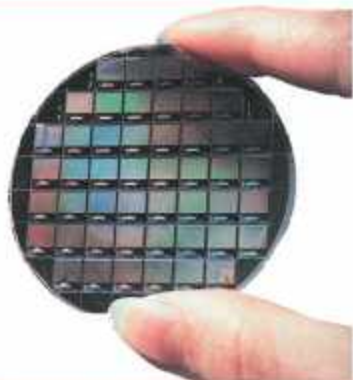
- ◆ Make reasonable estimates of value of physical quantities [of those quantities that are discussed in the topics of this grade].
- ◆ Use the conventions for indicating units, as set out in the SI units.
- ◆ Express derived units as products or quotients of the SI base units
- ◆ Analyze the homogeneity of physical equations [Through dimensional analysis]
- ◆ Derive formulae in simple cases [Through using dimensional analysis]
- ◆ Analyze and critique the accuracy and precision of data collected by measuring instruments
- ◆ Justify why all measurements contain some uncertainty.
- ◆ Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties]
- ◆ Quote answers with correct scientific notations, number of significant figures and units in all experimental and numerical results.

Physics is the most fundamental branch of physical sciences. It provides the basic principles and laws which help to understand the mysteries of other branches of sciences such as astronomy, chemistry, geology, biology and health sciences. The tools, techniques and products of Physics have transformed our dreams into realities. The comforts and pleasures added in our lives are fruitful results of science, technology and engineering in everyday life.

The information technology has entirely changed the outlook of mankind. The fast means of communication have brought people of the entire world in so close contact that the whole world has become a global village.

Physics is an experimental science and the scientific method emphasizes the need of accurate measurement of various measurable physical quantities. This chapter stresses in understanding the concept of measuring techniques and recording skills.

Think over!



Computer chips are made from silicon, which is obtained from sand. It is up to us whether we make a sand castle or a computer out of it.

1.1 PHYSICAL QUANTITIES AND THEIR UNITS

The foundation of physics depends on physical quantities in terms of which the laws of Physics are expressed. Therefore, these quantities have to be measured accurately. These are mass, length, time, velocity, force, density, temperature, electric current, and numerous others.

Physical quantities are often divided into two categories: base quantities and derived quantities. Derived quantities are those which depend on base quantities. Examples of derived quantities are velocity, acceleration, force, etc. Base quantities are not defined in terms of other physical quantities. The base quantities are the independent physical quantities in terms of which the other physical quantities can be defined. Typical examples of base quantities are length, mass and time.

The measurement of a base quantity involves two steps: first, the choice of a standard, and second, the establishment of a procedure for comparing the quantity to be measured with the standard so that a number and a unit are determined as the measure of that quantity.

Measurements must be reliable and accurate so that they can be used, easily and effectively.

1.2 INTERNATIONAL SYSTEM OF UNITS

In 1960, an international committee agreed on a set of definitions and standards to describe the physical quantities. The system that was established is called the System International (SI).

SI units are used by the world's scientific community and by almost all nations. The system international (SI) consists of two kinds of units: base units and derived units.

Base Units

There are seven base units for physical quantities namely: length, mass, time, temperature, electric current, light or luminous intensity and amount of substance (with special reference to the number of particles). Prefixes such as milli, micro, kilo, etc. may be used with them to express smaller or larger quantities.

The names of base units for these physical quantities together with symbols are listed in Table 1.1.

Areas of Physics

- Mechanics
- Heat & thermodynamics
- Electromagnetism
- Optics
- Sound
- Hydrodynamics
- Special relativity
- General relativity
- Quantum mechanics
- Atomic physics
- Molecular physics
- Nuclear physics
- Solid state physics
- Particle physics
- Superconductivity
- Superfluidity
- Plasma physics
- Magnetohydrodynamics
- Space physics

Interdisciplinary areas of Physics

- Astrophysics
- Biophysics
- Chemical physics
- Engineering physics
- Geophysics
- Medical physics
- Physical oceanography
- Physics of music

Table 1.1

Physical Quantity	SI Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd
Amount of substance	mole	mol

Derived Units

Derived units are those units which depend on the base units. Some of the derived units are given in Table 1.2. The units of plane angle and solid angle have also been included in the list of derived units since 1995.

In addition to base and derived units, the SI permits the use of certain additional units, including:

- The traditional mathematical units for measuring angles (degree, arcminute, and arcsecond).
- The traditional units for standard time are (minute, hour, day, and year).
- The logarithmic units bel (and its multiples, such as the decibel).
- Two metric units commonly used in ordinary life: the litre for volume and the tonne (metric ton) for large masses.
- Two non-metric scientific units are atomic mass unit (μ) and the electron volt (eV).
- The nautical mile and knot; units traditionally used at sea and in meteorology.
- The acre and hectare, common metric units of land area.
- The bar is a unit of pressure and it is commonly used as the millibar in meteorology and the kilobar in engineering.
- The angstrom and the barn, units used in physics and astronomy.

Scientific Notation

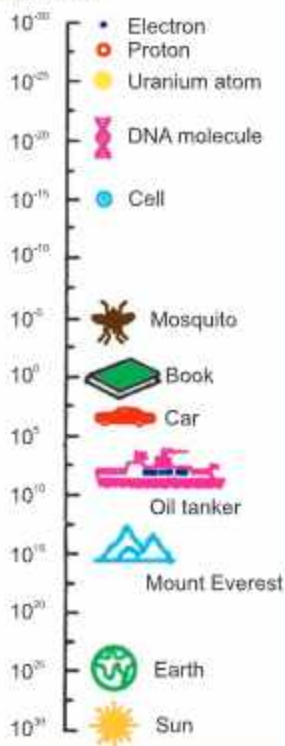
Numbers are expressed in standard form called scientific notation, which employs powers of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus, the number 134.7 should be written as 1.347×10^2 and 0.0023 be expressed as 2.3×10^{-3} .

Table 1.2

Physical quantity	Unit	Symbol	In terms of base units
Plane angle	radian	rad	dimensionless
Solid angle	steradian	sr	dimensionless
Force	newton	N	kg m s^{-2}
Work	joule	J	$\text{N m} = \text{kg m}^2 \text{s}^{-2}$
Power	watt	W	$\text{J s}^{-1} = \text{kg m}^2 \text{s}^{-3}$
Electric charge	coulomb	C	A s
Pressure	pascal	Pa	$\text{N m}^{-2} = \text{kg m}^{-1} \text{s}^{-2}$

Interesting information

Mass (kg)



Prefixes

Most prefixes indicate order of magnitude in steps of 1000 and provide a convenient way to express large and small numbers, to eliminate non-significant digits. SI also includes four of the other prefixes to accommodate usage already established before the introduction of SI (Table 1.3). They are centi-(10^{-2}), deci-(10^{-1}), deka-(10^1) and hecto-(10^2).

Conventions for Using SI Units

Use of SI units requires special care, more particularly in writing prefixes. Some points to note are:

1. Each SI unit is represented by a symbol not an abbreviation. These symbols are the same in all languages. Hence, correct use of the symbol is very important.

For example: For ampere, we should use "A" not "amp"; for seconds, "s" not "sec", SI not S.I.

2. Full name of unit does not begin with capital letter.

For example: newton, metre, etc., except Celsius.

3. Symbols appear in lower case.

For example: "m" for metre, "s" for second, etc., exception "L" for litre.

4. Symbols named after scientists have initial letters capital.

For example: "N" for newton, "Pa" for pascal, "W" for watt.

5. Symbols and prefixes are printed in upright (roman) style regardless of the type style in surrounding text.

For example: a distance of 50 m.

6. Symbols do not take plural form.

For example: 1mm, 100 mm, 1 kg, 60 kg.

7. No fullstop or dot is placed after the symbol except at the end of the sentence.

8. Prefix is written before and without space to base unit.

For example: "mL" not m L or "ms" not m s.

9. Base units are written one space apart. Leave a space even between the number (value) and the symbol.

For example: 1 kg, 10 m s⁻¹, etc.

Table 1.3
Some Prefixes for Powers of Ten

Factor	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^1	deca	da
10^2	hecto	h
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

For your information

	Interval (s)
Age of the universe	5×10^{17}
Age of the Earth	1.4×10^{17}
One year	3.2×10^7
One day	8.6×10^4
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radiowaves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

Approximate Values of Some Time Intervals

Do you know?

Mass can be thought of as a form of energy. In effect, the mass is highly concentrated form of energy. Einstein's famous equation, $E=mc^2$ means:

$$\text{Energy} = \text{mass} \times (\text{speed of light})^2$$

According to this equation 1 kg mass is actually 9×10^{16} J of energy.

10. Compound prefixes are not allowed:

For example: $1\mu\mu\text{F}$ should be 1 pF .

11. When base unit of multiple is raised to a power, the power applies to whole multiple and not to base unit alone.

For example: $1\text{ km}^2 = 1(\text{km})^2 = 1 \times (10^3\text{ m})^2 = 1 \times 10^6\text{ m}^2$.

12. Use negative index notation (m s^{-1}) instead of solidus (m/s).

13. Use scientific notation, that is, one non-zero digit left of decimal.

For example: $143.7 = 1.437 \times 10^2$.

14. Do not mix symbols and names in the same expression.

For example: metre per second or m s^{-1} , not metre/sec or m/second.

15. Practical work should be recorded in most convenient units depending upon the instruments being used.

For Example: Measurements using screw gauge should be recorded in mm but the final results must be recorded to the appropriate base units.

16. System International do not allow the use of former CGS System units such as dyne, erg, gauss, poise, torr, etc.

1.3 UNCERTAINTY IN MEASUREMENT

You can count the number of pages of a book exactly but measurement of its length needs some measuring instrument. Every instrument is calibrated to a certain smallest division mark on it and this fact puts a limit regarding its accuracy. When you take a reading with one instrument, its limit of measurement is the smallest division or graduation on its scale. Hence, every measured quantity has some uncertainty about its value. When a measurement is made, it is taken to the nearest graduation or marking on the scale. You can estimate the maximum uncertainty as being one smallest division of the instrument. This is called absolute uncertainty. It is one millimetre on a metre rule that is graduated in millimetres. For example, if one edge of the book coincides with 10.0 cm mark and the other with 33.5 cm, then the length with uncertainty is given by

$$(33.5 \pm 0.05)\text{ cm} - (10.0 \pm 0.05)\text{ cm} = (23.5 \pm 0.1)\text{ cm}$$

It means that the true length of the book is in between 23.4 cm and 23.6 cm. Hence, the maximum uncertainty is ± 0.05 cm, which is equivalent to an uncertainty of 0.1 cm. Infact, it is equal to least count of the metre rule. Uncertainty may be recorded as:

$$\text{Fractional uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}}$$

$$\text{or Percentage uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}} \times 100\%$$

Uncertainty in Digital Instruments

Some modern measuring instruments have a digital scale. We usually estimate one

digit beyond what is certain: with a digital scale, this is reflected in some fluctuations of the last digit. If the last digit fluctuates by 1 or 2, write down that last digit. If fluctuation is more than 2 or so in the last digit, it may mean that the reading is being influenced by some factor such as air currents. Regardless of the reason, a large fluctuation may mean that the displayed digit is not really significant.

The indication of uncertainty in a recorded value has been simplified using significant figures. If a measurement is recorded using the knowledge of significant figures, then its last digit, which is an estimation, is an indication of the accuracy of the recorded value.

1.4 USE OF SIGNIFICANT FIGURES

The number of digits of a measurement about which we do feel reasonably sure are called significant figures. Infact, they reflect the use of actual instrument used for that measurement. While using a calculator, the result of any calculation contains many digits after the decimal point. The additional digits may mislead another person who uses those figures into believing them. Hence, they are to be rounded off to the correct number of significant figures. This can be done by keeping in view the uncertainty or the least count of the instrument while recording observations and also quoting results of any calculations to the correct numbers of significant figures. It is better to quote the result in scientific notation to avoid any ambiguity regarding the number of significant figures. For example, weighing the same object with different balances:

Electronic balance : mass = 3.145 ± 0.001 g

Lever balance : mass = 3.1 ± 0.1 g

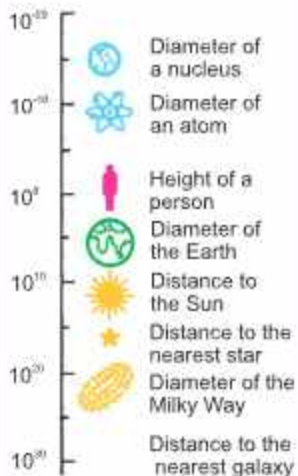
Usually, the uncertainty ± 0.001 g or ± 0.1 g is dropped, and it is understood that the number quoted has an uncertainty of at least 1 unit in the last digit. All digits which are quoted are called significant figures.

In any measurement, the accurately known digits and the first estimated or doubtful digit are called significant figures.

Proper use of significant figures ensures that we correctly represent the uncertainty of our measurements. For example, scientists immediately realize that the reported mass 3.145 g is more accurate than a reported mass of 3.1 g, reflecting the use of a better or more precise instrument. As we improve the quality of our measuring instrument and techniques, we extend the result to more and more significant figures and

For your information

Distance (m)



Order of magnitude of some distances

correspondingly improve the experimental accuracy of the result.

Working with significant figures

(i) Counting significant digits

- All digits 1,2,3,4,5,6,7,8,9 are significant. However, zeros may or may not be significant. In case of zeros, the following rules may be adopted:
- A zero between two significant figures is itself significant.
- Zeros to the left most significant figure are not significant. For example, none of the zeros in 0.00467 or 02.59 are significant.
- Zeros to the right of a significant figure may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8,000 kg, the number of significant zeros is determined by the precision of the measuring instrument. If the measuring scale has a least count of 1 kg, then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg and so on.
- When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example, a measurement recorded as 8.70×10^4 kg has three significant figures.

(ii) Multiplying or dividing numbers

Keep a number of significant figures in the product or quotient not more than that contained in the least accurate factor i.e., the factor containing the least number of significant figures. For example, the computation of the following using a calculator, gives

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^4}{1.336} = 1.45768982 \times 10^3$$

As the factor 3.64×10^4 , the least accurate in the above calculation has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off for which the following rules are followed:

- If the first digit dropped is less than 5, the last digit retained should remain unchanged.
- If the first digit dropped is more than 5, the digit to be retained is increased by one.
- If the digit to be dropped is 5, the previous digit which is to be retained is increased by one if it is odd and retained as such if it is even. For example, the following numbers are rounded off to three significant figures as follows. The digits are deleted one by one.

Remember Thumb Rule

For calculation of end result:

- Addition / Subtraction: same precision.
- Multiplication / Division: same accuracy (Same number of significant figures).

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.9
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Following this rule, the correct answer of the computation given in section (ii) is 1.46×10^3 .

(iii) In adding or subtracting numbers

The number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters. For example, suppose we wish to add the following quantities expressed in metres.

(a)	72.1	(b)	2.7543
	3.42		4.10
	<u>0.003</u>		<u>1.273</u>
	75.523		8.1273

Correct answer: 75.5 m 8.13 m

In case (a), the number 72.1 has the smallest decimal places, thus the answer is rounded off to the same position which is then 75.5 m. In case (b), the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal position which is then 8.13 m.

Limitations of Significant Figures

Significant figures deal with only one source of uncertainties that inherent in reading the scale. Real experimental uncertainties have many contributions, including personal errors and sometimes hidden systematic errors. One cannot do better than that what the scale reading allows, but the total uncertainty may well be more than what the significant figure of the measurements would suggest.

1.5 PRECISION AND ACCURACY

The terms precision and accuracy are frequently used in physics measurements. They should be distinguished clearly. The precision of a measurement is determined by the instrument or device being used. The smaller the least count the more precise is the measurement. Accuracy is defined

as the closeness of a measurement to the exact or accepted value of a physical quantity. It is expressed by the fractional or percentage uncertainty. The smaller the fractional or

Thumb Rule for Uncertainty

For average value of many readings:

- Mean deviation from an average value.
- **Periodic Uncertainty:**
Divide least count of timing device by the number of oscillations.

Quick Quiz

1. Give the correct number of significant figures for 0.0054 m, 0.03030 m, 40.0 m, 0.5 m, 8.20×10^3 m.
2. Give the answer to the appropriate number of significant figures.
 $2602 \text{ kg} + 36.02 \text{ kg} + 54.1 \text{ kg} = ?$
3. Give the answer to the appropriate number of significant figures.
 $3.54 \text{ kg} - 2.4 \text{ kg} = ?$
4. Give the answer to the appropriate number of significant figure.
 $2.45 \times 10^3 \text{ m} \times 2.46 \text{ m} / 3.6 \text{ m} = ?$

Remember Thumb Rule

- Precision: Less absolute uncertainty.
- Accuracy: Less % age uncertainty.

percentage uncertainty, the more accurate is the measurement.

For example, the length of an object is recorded as 25.5 cm by using a metre rule having smallest division in millimetre. Its precision or absolute uncertainty (least count) = ± 0.1 cm.

$$\text{Fractional uncertainty} = \frac{0.1 \text{ cm}}{25.5 \text{ cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1 \text{ cm}}{25.5 \text{ cm}} \times 100\% = 0.4\%$$

Another measurement taken by Vernier Callipers with least count 0.01 cm is recorded as 0.45 cm. It has precision or absolute uncertainty (least count) = ± 0.01 cm.

$$\text{Fractional uncertainty} = \frac{0.1 \text{ cm}}{0.45 \text{ cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.1 \text{ cm}}{0.45 \text{ cm}} \times 100\% = 2\%$$

Thus, the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by Vernier Callipers is more precise but is less accurate. In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometer screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that:

A precise measurement is the one which has less precision or absolute uncertainty and an accurate measurement is the one which has less fractional or percentage uncertainty.

We can never make an exact measurement. The best we can do is to come as close as possible within the limitation of the measuring instrument.

1.6 ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT

Knowing the uncertainties in all the factors involved in a calculation, the maximum possible uncertainty or error in the final result can be found as follows:

For your information



We use many devices to measure physical quantities, such as length, time, and temperature. They all have some limit of precision.

For your information



These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Accademia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.

1. For addition and Subtraction

Absolute uncertainties are added. For example, the distance between two positions $x_1 = 15.4 \pm 0.1$ cm and $x_2 = 25.6 \text{ cm} \pm 0.1$ cm is recorded as:

$$x = x_2 - x_1 = 10.2 \pm 0.2 \text{ cm}$$

and addition of two lengths is:

$l_1 = 8.5 \pm 0.1$ cm and $l_2 = 12.6 \pm 0.1$ cm recorded as:

$$l = l_1 + l_2 = 21.1 \pm 0.2 \text{ cm}$$

2. For multiplication and division

Percentage uncertainties are added. For example, the maximum possible uncertainty in the value of resistance R of a conductor determined by the potential difference V applied across the conductor resulting in current flowing through it is estimated as under:

$$\text{Let } V = 3.4 \pm 0.1 \text{ V}$$

$$I = 0.68 \pm 0.05 \text{ A}$$

$$\text{using } R = \frac{V}{I}$$

$$\text{Percentage uncertainty in } V = \frac{0.1 \text{ V}}{3.4 \text{ V}} \times 100\% = 3\%$$

$$\text{Percentage uncertainty in } I = \frac{0.05 \text{ A}}{0.68 \text{ A}} \times 100\% = 7\%$$

Hence, total percentage uncertainty in the value of R is $3 + 7 = 10\%$

The value of R will be written as:

$$R = \frac{3.4 \text{ V}}{0.68 \text{ A}} = 5.0 \text{ ohm}$$

Hence, $R = 5.0 \pm 0.5$ ohms, uncertainty being an estimate only, is recorded by one significant figure.

3. For Power Factor

The percentage uncertainty is multiplied by the power factor in the formula. For example, the calculation of cross-sectional area of a cylinder of radius $r = 1.25$ cm using formula for Area $A = \pi r^2$ is given by the %age uncertainty which is $A = 2 \times$ %age uncertainty in radius r . As uncertainty is multiplied by power factor, it increases the precision demand of measurement. When the radius of a small sphere is measured as 1.25 cm by Vernier Callipers with least count 0.01 cm, then

$$\text{The radius } r \text{ is recorded as } r = 1.25 \pm 0.01 \text{ cm}$$

$$\% \text{age uncertainty in radius } r \text{ is } r = \frac{0.01}{1.25} \times 100\% = 0.8\%$$

$$\text{Total percentage uncertainty in area } A = 2 \times 0.8 = 1.6\%$$

Thus

$$A = \pi r^2 \\ = 3.14(1.25)^2 = 4.906 \text{ cm}^2 \text{ with } 1.6\% \text{ uncertainty}$$

For your information

Colour printing uses just four colours—cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

Thumb Rule for Total Uncertainty

- **For addition and subtraction:**
Absolute uncertainties are added.
- **For multiplication and division:**
Percentage uncertainties are added.
- **For power factor:**
Power factor \times Percentage uncertainty

For your information

Travel time of light

Moon to Earth	1 min 20 s
Sun to Earth	8 min 20 s
Pluto to Earth	5 h 20 s

Thus, the result should be recorded as $A = 4.91 \pm 0.08 \text{ cm}^2$

Example 1.1 The length, breadth and thickness of a metal sheet are 2.03 m, 1.22 m and 0.95 cm respectively. Calculate the volume of the sheet correct up to the appropriate significant digits.

Solution	Given Length	$\ell = 2.03 \text{ m}$
	Breadth	$b = 1.22 \text{ m}$
	Thickness	$h = 0.95 \text{ cm} = 0.95 \times 10^{-2} \text{ m}$
	Volume	$V = \ell \times b \times h = 2.03 \text{ m} \times 1.22 \text{ m} \times 0.95 \times 10^{-2} \text{ m}$ $= 2.35277 \times 10^{-2} \text{ m}^3$

As the factor 0.95 cm has minimum number of significant figures equal to two, therefore, volume is recorded up to 2 significant figures, hence, $V = 2.4 \times 10^{-2} \text{ m}^3$

Example 1.2 The mass of a metal box measured by a lever balance is 3.25 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct up to the appropriate precision?

Solution

$$\begin{aligned} \text{Total mass when silver coins are added to box} &= 3.25 \text{ kg} + 0.01001 \text{ kg} + 0.01002 \text{ kg} \\ &= 3.27003 \text{ kg} \end{aligned}$$

Since least precise mass is 3.25 kg, having two decimal places, hence, total mass should be reported to 2 decimal places which is the appropriate precision.

Thus Total mass = 3.27 kg

Example 1.3 The diameter and length of a metal cylinder measured with the help of Vernier Callipers of least count 0.01 cm are 1.25 cm and 3.35 cm, respectively. Calculate the volume V of the cylinder and uncertainty in it.

Solution

Given	
Diameter	$d = 1.25 \text{ cm}$ with least count 0.01 cm
Length	$\ell = 3.35 \text{ cm}$ with least count 0.01 cm
Absolute uncertainty in length	$= 0.01 \text{ cm}$
%age uncertainty in length	$= (0.01 \text{ cm} / 3.35 \text{ cm}) \times 100\% = 0.3\%$
Absolute uncertainty in diameter	$= 0.01 \text{ cm}$
%age uncertainty in diameter	$= (0.01 \text{ cm} / 1.25 \text{ cm}) \times 100\% = 0.8\%$
As	Volume $= \pi r^2 \ell = \pi \frac{d^2}{4} \ell$

Total uncertainty in $V = 2$ (%age uncertainty in diameter) + (%age uncertainty in length)
 $= 2 \times 0.8\% + 0.3\% = 1.9\%$

Then $V = 3.14 \times (1.25 \text{ cm})^2 \times 3.35 \text{ cm} / 4 = 4.1089842 \text{ cm}^3$ with 1.9% uncertainty

Thus $V = (4.11 \pm 0.08) \text{ cm}^3$

where 4.11 cm^3 is calculated volume and 0.08 cm^3 is the uncertainty in it.

1.7 DIMENSIONS OF PHYSICAL QUANTITIES

Any physical quantity can be described by certain familiar properties such as length, mass, time, temperature, electric current, etc. These measurable properties are called dimensions. Dimensions deal with the qualitative nature of a physical quantity in terms of fundamental quantities. The quantities such as length, depth, height, diameter, light year are all measured in metre and denoted by the same dimension, basically known as length given by symbol L written within square brackets $[L]$. Similarly, the other fundamental quantities, mass, time, electric current and temperature are denoted by specific symbols $[M]$, $[T]$, $[A]$ and $[\theta]$, respectively. These five dimensions have been chosen as being basic because they are easy to measure in experiments.

The dimensions of other quantities indicate how they are related to the basic quantities and are combination of fundamental dimensions. For example, speed v is measured in metres per second, so it has the dimensions of length $[L]$ divided by time $[T]$.

$$[v] = [L]/[T] = [L][T^{-1}] = [LT^{-1}]$$

As the acceleration $a = \Delta v / \Delta t$

Dimensions of acceleration are

$$[a] = [v]/[T] = [LT^{-1}]/[T] = [LT^{-2}]$$

Also, dimensions of force can be written as

$$[F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

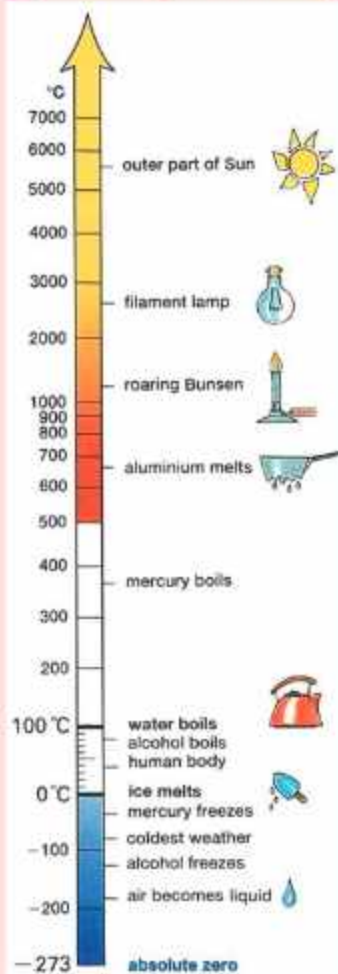
By the use of dimensionality, we can check the homogeneity (correctness) of a physical equation, and also, we can derive formula for a physical quantity.

Homogeneity of Physical Equations

The correctness of an equation can be checked by showing that the dimensions of quantities on both sides of the equation are the same. This is known as principle of homogeneity.

Suppose a car starts from rest ($v_i = 0$) and covers a distance S in time t moving with an acceleration a . The

Interesting information



Some Specific Temperatures

For your information



Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

equation of motion is given by,

$$S = v_1 t + \frac{1}{2} a t^2$$

or

$$S = \frac{1}{2} a t^2$$

Numerical factors like 1/2 have no dimensions, so they can be ignored. By putting the dimensions of both sides of the equation:

$$[S] = [a][t^2]$$

Writing the symbols of dimensions $[L] = [LT^{-2}][T^2]$

$$[L] = [LT^{-2}T^2]$$

or

$$[L] = [L]$$

This shows that dimensions on both sides of equation are the same, therefore, the equation is dimensionally correct.

Derivation of a Formula

Dimensionality can be used to derive a possible formula for a physical quantity by correct estimation of various factors on which the quantity depends.

Example 1.4

Derive a formula for the centripetal force required to keep an object moving along a circle with uniform speed. Assuming that centripetal force depends on mass of the object, radius of the circle and uniform speed.

Solution

As force depends on mass m of the object, radius r of the circle and uniform speed v , we can write:

$$F \propto m^a v^b r^c$$

$$F = (\text{constant}) m^a v^b r^c \quad \dots\dots\dots (i)$$

where the exponents (powers) a , b and c are to be determined. By the principle of homogeneity, the dimensions on both sides of the equation should be the same. Since, constant has no dimension so by ignoring it, we write the above equation in terms of dimensions as,

$$[F] = [m^a][v^b][r^c]$$

$$[MLT^{-2}] = [M^a][L/T]^b [L]^c$$

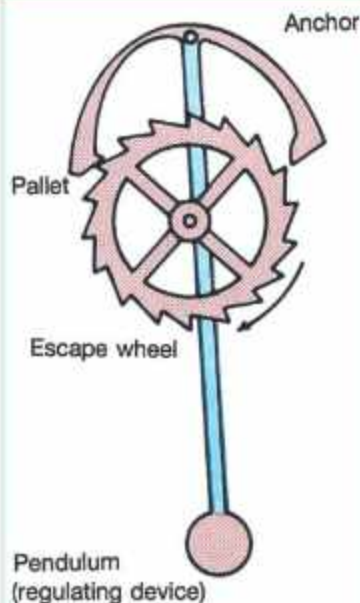
$$[MLT^{-2}] = [M^a][LT^{-1}]^b [L]^c$$

$$[MLT^{-2}] = [M^a L^{b+c} T^{-b}] \quad \dots\dots\dots (ii)$$

Comparing the powers of dimensions on both sides of the above equation, we have

$$a = 1$$

Do you know?



The device which made the pendulum clock practical.

Beware!

Calculators are designed to yield as many digits as the memory of the calculator chip permits. Hence, be sure to round off the final answers of calculations down to correct number of significant figures.

$$b + c = 1$$

$$-b = -2$$

Solving the above equations, we have $a = 1$, $b = 2$, $c = -1$

Putting the values of a , b and c in equation (i), we have

$$F = (\text{constant}) mv^2 r^{-1}$$

or

$$F = (\text{constant}) mv^2 / r$$

The numerical value of the constant cannot be determined by dimensional analysis. However, it can be found by experiments. In the above equation, numerical value of the constant happens to be "1", so the equation reduces to:

$$F = mv^2 / r$$

Limitations in Dimensional Analysis

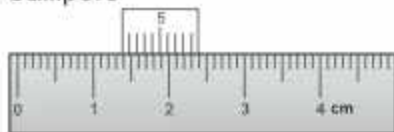
The dimensional method cannot identify where an equation is wrong. Even if an equation is proved correct, we can only say the equation might be correct, for the reason that the method does not provide a check on any numerical factor or constant. That can only be determined by experiments or plotting some suitable graph between the variables.

QUESTIONS

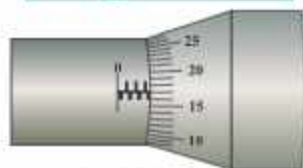
Multiple Choice Questions

Tick (✓) the correct answer.

- 1.1 The purpose of study and discoveries in Physics is:
- the probing of interstellar spaces
 - the betterment of mankind
 - the development of destructive technology in warfare
 - development in aesthetics for the world
- 1.2 The length of a steel pipe is in between 0.7 m to 0.8 m. Identify from the following, the appropriate instrument to be used for an accuracy of 0.001 m.
- A micrometer screw gauge
 - A metre rule
 - A ten metres measuring tape
 - A Vernier Callipers
- 1.3 The diameter of a steel ball is measured using a Vernier Callipers and its reading is shown in the figure. What is the diameter of the steel ball?
- 1.30 cm
 - 1.39 cm
 - 1.40 cm
 - 1.31 cm



- 1.4 The figure shows the reading on a micrometer screw gauge used to measure diameter of a thin rod. One complete turn of the thimble is 0.50 mm and there are 50 lines on the circular scale. The diameter of the rod is:



- (a) 3.67 mm (b) 3.17 mm (c) 4.17 mm (d) 4.20 mm
- 1.5 The number of significant figures of a measurement are defined as:
- (a) they reflect the accuracy of the observation in a measurement
 (b) they are the figures which are reasonably reliable
 (c) they are the accurately known digits and the first doubtful digit of a measurement
 (d) all of the above
- 1.6 The number of significant figures in the measured mass 2500.0 kg is:
- (a) two (b) three (c) four (d) five
- 1.7 The sum $12 \text{ kg} + 2.02 \text{ kg} + 5.1 \text{ kg}$ according to appropriate precision is:
- (a) 19 kg (b) 19.0 kg (c) 19.1 kg (d) 19.12 kg
- 1.8 The answer to appropriate precision for the subtraction $(1.126 - 0.97268)$ is:
- (a) 0.15 (b) 0.153 (c) 0.1533 (d) 0.15332
- 1.9 The answer of the product (2.8723×1.6) to the appropriate number of significant figures is:
- (a) 4.59568 (b) 4.595 (c) 4.59 (d) 4.6
- 1.10 The answer to the mathematical division $(45.2 \div 6.0)$ in appropriate number of significant figures is:
- (a) 7.5 (b) 7.53 (c) 7.533 (d) 7.5333
- 1.11 The answer to the following mathematical operation $24.4 \text{ m} \times 100 \text{ m} / 5.0 \text{ m}$ to the appropriate number of significant figures is:
- (a) 4880 m (b) 4900 m (c) $4.88 \times 10^3 \text{ m}$ (d) $4.9 \times 10^2 \text{ m}$
- 1.12 The ratio of the dimensions of force and energy is:
- (a) T (b) T^{-1} (c) L (d) L^{-1}
- 1.13 Identify which pair from the following does not have identical dimensions.
- (a) Work and torque
 (b) Angular momentum and Planck's constant
 (c) Moment of inertia and moment of force
 (d) Impulse and momentum
- 1.14 The following figures are of the same Vernier Callipers. Figure (1) shows the reading when the jaws are closed while



Fig. (2) shows the reading when a solid cylinder is placed between the jaws. The length of the cylinder is:

- (a) 3.26 cm (b) 3.30 cm (c) 3.34 cm (d) 4.20 cm

1.15 The least count of an instrument determines:

- (a) precision of a measurement
(b) accuracy of a measurement
(c) fractional uncertainty of a measurement
(d) percentage uncertainty of a measurement

1.16 A measuring tape has been graduated with a minimum scale division of 0.2 cm. The allowed reading using this tape may be:

- (a) 80.5 cm (b) 80.6 cm (c) 80.65 cm (d) 80.7 cm

Short Answer Questions

- 1.1 What are base units and derived units? Give some examples of both these units.
- 1.2 How many significant figures should be retained in the following?
(i) Multiplying or dividing several numbers (ii) Adding or subtracting numbers
- 1.3 How is the Vernier scale related to the main scale of a Vernier Callipers?
What is meant by L.C. of the Vernier Callipers?
- 1.4 Write the following numbers in scientific notation:
a) 143.7 (b) 206.4×10^2
- 1.5 Write the following numbers using correct prefixes:
(a) $580 \times 10^2 \text{ g}$ (b) $0.45 \times 10^{-5} \text{ s}$
- 1.6 Kinetic energy of a body of mass m moving with speed v is given by $\frac{1}{2}mv^2$. What are the dimensions of kinetic energy?
- 1.7 How many significant figures are there in the following measurements?
(i) 37 km (ii) 0.002953 m (iii) 7.50034 cm (iv) 200.0 m
- 1.8 Write the dimensions of: (i) Planck's constant (ii) angular velocity

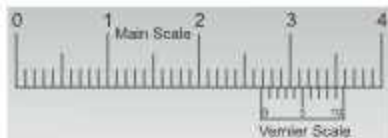
Constructed Response Questions

- 1.1 Why do we find it useful to have two units for the amount of a substance, the kilogram and the mole?
- 1.2 Three students measured the length of a rod with a scale on which minimum division is 1 mm and recorded as: (i) 0.4235 m (ii) 0.42 m (iii) 0.424 m. Which record is correct and why?
- 1.3 Why is the kilogram (not the gram), the base unit of mass.
- 1.4 Consider the equation; $P = Q + R$
If Q and R both have the dimensions of $[MLT]$, what are the dimensions of P ? What

are the units of P in SI? If the dimensions of Q were different from those of R , could we determine dimensions of P ?

- 1.5 What is the least count of a clock if it has:
- Hour's hand, minute's hand and second's hand
 - Hour's hand and minute's hand
- 1.6 How can the diameter of a round pencil be measured using metre rule with the same accuracy as that of Vernier Callipers? Describe.
- 1.7 How would be the readings differ if the screw gauge is used instead of a Vernier Callipers to measure the thickness of a glass plate?

- 1.8 Write the correct reading of the length of a solid cylinder as shown in the figure if there is an error of $+0.02$ cm in the Vernier Callipers.

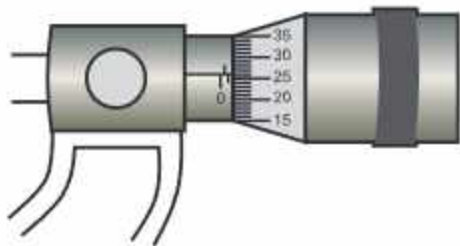


- 1.9 There are 50 divisions on the circular scale of a screw gauge. If the head (thimble) of the screw is given 10 revolutions, then the spindle advances by 5 mm. There is also zero error as the 2nd division of the circular scale coincides with the datum line and zero of circular scale is below the datum line. What is the thickness of a glass slab as measured by the described screw gauge shown in the figure?



- 1.10 What is meant by a dimensionless quantity? Give one example.

- 1.11 A student uses a screw gauge to determine the thickness of a sheet of paper. The student folds the paper three times and measures the total thickness of the folded sheet. Assume that there is no zero error in the screw gauge. The reading of screw gauge is shown in the figure. Find the thickness of the sheet.



- 1.12 Round off each of the following numbers to 3 significant figures and write your answer in scientific notation.

(a) 0.02055 (b) 4656.5

Comprehensive Questions

- 1.1 What is meant by uncertainty in a measurement? How the uncertainty in a digital instrument is indicated?
- 1.2 Differentiate between the terms precision and accuracy with reference to measurement of physical quantities.
- 1.3 (a) What is meant by significant figures? Write two reasons for using them in measurements. How to find the uncertainty in a timing experiment such as the

time period of a simple pendulum?

- (b) The mass of a solid cylinder is 12.85 g. Its length is 3.35 cm and diameter is 1.25 cm. Find the density of its material expressing the uncertainty in the density.
- 1.4 Explain with examples the writing of physical quantities into their dimensions. Write its two benefits.
- 1.5 Check the homogeneity of the relation:

$$v = \sqrt{\frac{T \times \ell}{m}}$$

where v is the speed of transverse wave on a stretched string of tension T , length ℓ and mass m .

Numerical Problems

- 1.1 Astronomers usually measure astronomical distances in light years. One light year is the distance that light travels in one year. If speed of light is $3 \times 10^8 \text{ m s}^{-1}$, what is one light year in metres? (Ans: $9.5 \times 10^{15} \text{ m}$)
- 1.2 Write the estimated answer of the following in standard form.
- (a) How many seconds are there in 1 year?
- (b) How many years are in 1 second? (Ans: (a) $3.2 \times 10^7 \text{ s}$ (b) $3.1 \times 10^{-8} \text{ years}$)
- 1.3 The length and width of a rectangular plate are measured to be 18.3 cm and 14.60 cm, respectively. Find the area of the plate and state the answer to correct number of significant figures. (Ans: 267 cm^2)
- 1.4 Find the sum of the masses given in kg up to appropriate precision:
- (i) 3.197 (ii) 0.068 (iii) 13.9 (iv) 3.28 (Ans: 20.4 kg)
- 1.5 The diameter and length of a metal cylinder measured with the help of a Vernier Callipers of least count 0.01 cm are 1.22 cm and 5.35 cm respectively. Calculate its volume and uncertainty in it. (Ans: $6.2 \pm 0.1 \text{ cm}^3$)
- 1.6 Show that the expression; $v_f^2 - v_i^2 = 2aS$ is dimensionally correct, where v_i is the initial velocity, a is the acceleration and v_f is the velocity after covering a distance S .
- 1.7 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.
- 1.8 Derive a formula for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period T may depend are:
- (i) length of the pendulum ℓ
- (ii) mass of the bob m
- (iii) angle θ which the thread makes with the vertical
- (iv) acceleration due to gravity g . (Ans: $T = \text{constant} \sqrt{\frac{\ell}{g}}$)

Force and Motion

Learning Objectives

After studying this chapter, the students will be able to:

- ◆ Differentiate between scalar and vector quantities
- ◆ Represent a vector in 2-D as two perpendicular components.
- ◆ Describe the product of two vectors (dot and cross-product) along with their properties.
- ◆ Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically]
- ◆ Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion]
- ◆ Evaluate and analyse projectile motion in the absence of air resistance
[This includes solving problems making use of the below facts:
 - (i) Horizontal component (V_x) of velocity is constant.
 - (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.
 - (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:
 - How high does it go?
 - How far would it go along the level land?
 - Where would it be after a given time?
 - How long will it remain in flight?
 Situations may also require students to calculate for a projectile launched from ground height the
 - launch angle that results in the maximum range.
 - relation between the launch angles that result in the same range.]
- ◆ Predict qualitatively how air resistance affects projectile motion. [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- ◆ Apply the principle of conservation of momentum to solve simple problems [Including elastic and inelastic interactions between objects in both one and two dimensions.
Knowledge of the concept of coefficient of restitution is not required.
Examples of applications include:
 - karate chops to break a pile of bricks
 - car crashes
 - ball & bat
 - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- ◆ Predict and analyse motion for elastic collisions [This includes making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- ◆ Justify how the momentum of a closed system is always conserved, some change in kinetic energy may take place.

BASIC CONCEPT OF SCALARS AND VECTORS

Scalars and vectors are basic concepts in physics. Many problems in physics require to distinguish between scalar and vector quantities to apply the correct mathematical and conceptual approaches. Understanding scalars and vectors help us to grasp how Physics applies to real-world situations, such as calculating the total distance travelled (scalar) or determining the magnitude and direction of force (vector). Learning these concepts develops critical thinking and problem-solving skills. This chapter is primarily concerned with vector algebra and its application in uniform accelerated motion, in a straight line, motion of freely falling bodies in uniform gravitational field, projectile motion, and interaction between objects in one and two dimensions.

2.1 SCALARS

Scalars are physical quantities that are described solely by a magnitude (size or amount) without any mention of direction. Thus, scalars are directionless and can be fully characterized by a single number and its associated unit.

Examples:

Mass: The amount of matter in an object. For example, 2 kg.

Distance: The total length of the path travelled by an object irrespective of the direction. For example, 50 m.

Speed: The rate at which an object covers distance. For example, 40 km h^{-1} .

Time: The duration between two events taking place. For example, 20 s.

Energy: The capacity to do work. For example, 25 J.

Temperature: A measure of the average kinetic energy of particles in a substance. For example, 20°C .

2.2 VECTORS

Those physical quantities which require magnitude as well as direction for their complete specification are known as vectors.

Examples:

Displacement: The change in position of an object. It has length, a distance (magnitude) and a direction (e.g. 10 m towards west).

Velocity: The speed of an object in a particular direction (e.g., 50 km h^{-1} towards west).

Acceleration: The rate of change of velocity that occurs in either speed or direction or both (e.g., 10 m s^{-2} upward).

Force: A push or pull acting on an object, determined by its magnitude and direction (e.g; 20 N to the right)

Graphical Representation of a Vector

A good way to represent a vector quantity is to use a vector diagram, in which vectors are often represented by arrows. The length of the arrows indicates the magnitude and the head of the arrow shows the direction of the vector. Vectors are typically denoted by bold face letters (e.g; \mathbf{V} , \mathbf{F}) or an arrow above symbol (\vec{A}).

Rectangular Components of a Vector

A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its component vectors along the specified directions. It is usually convenient to resolve a vector into its components along the mutually perpendicular directions. Such components are called rectangular components.

Let there be a vector \mathbf{A} represented by a line OP making an angle θ with the x -axis. Draw projection OM of vector \mathbf{A} on x -axis and projection ON of vector \mathbf{A} on y -axis as shown in Fig.2.1. Projection OM being along x -direction is represented by \mathbf{A}_x and projection ON along y -direction is represented by \mathbf{A}_y . By applying head to tail rule:

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y \quad \dots\dots\dots (2.1)$$

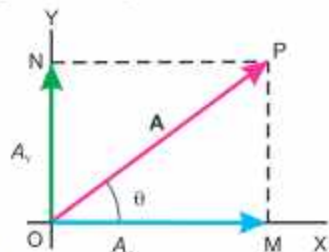


Fig. 2.1

Thus, \mathbf{A}_x and \mathbf{A}_y are the components of vector \mathbf{A} . Since these are at right angles to each other, they are called rectangular components of \mathbf{A} . Considering the right angled triangle OMP , the magnitude of \mathbf{A}_x or x -component of \mathbf{A} is:

$$A_x = A \cos \theta \quad \dots\dots\dots (2.2)$$

And the magnitude of \mathbf{A}_y or y -component of \mathbf{A} is:

$$A_y = A \sin \theta \quad \dots\dots\dots (2.3)$$

Determination of a Vector from its Rectangular Components

If the rectangular components of a vector as shown in Fig.(2.1) are given, we can find out the magnitude of the vector by using Pythagorean Theorem.

In the right angle $\triangle OMP$

$$(OP)^2 = (OM)^2 + (MP)^2$$

$$\text{or} \quad A^2 = A_x^2 + A_y^2 \quad \dots\dots\dots (2.4)$$

$$\text{or} \quad A = \sqrt{A_x^2 + A_y^2}$$

$$\text{The direction } \theta \text{ is given by} \quad \tan \theta = \frac{MP}{OM} = \frac{A_y}{A_x}$$

$$\text{or} \quad \theta = \tan^{-1} \left(\frac{A_y}{A_x} \right) \quad \dots\dots\dots (2.5)$$

Example 2.1 Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Solution Let θ be the angle between two forces F_1 and F_2 , where F_1 is along the x-axis. Then x-component of their resultant will be:

$$R_x = F_1 \cos 0^\circ + F_2 \cos \theta$$

$$R_x = F_1 + F_2 \cos \theta$$

And y-component of their resultant is

$$R_y = F_1 \sin 0^\circ + F_2 \sin \theta$$

$$R_y = F_2 \sin \theta$$

The resultant R is given by

$$R^2 = R_x^2 + R_y^2$$

As

$$R = F_1 = F_2 = F$$

Hence

$$F^2 = (F + F \cos \theta)^2 + (F \sin \theta)^2$$

$$F^2 = F^2 + F^2 \cos^2 \theta + 2F^2 \cos \theta + F^2 \sin^2 \theta$$

or

$$0 = 2F^2 \cos \theta + F^2 (\cos^2 \theta + \sin^2 \theta)$$

or

$$0 = 2F^2 \cos \theta + F^2, \quad 0 = F^2 (2 \cos \theta + 1)$$

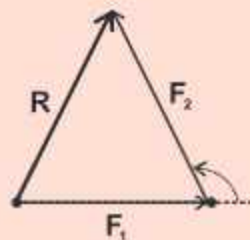
As $F \neq 0$, so $2 \cos \theta + 1 = 0$

or

$$\cos \theta = -0.5$$

or

$$\theta = \cos^{-1}(-0.5) = 120^\circ$$



2.3 PRODUCT OF TWO VECTORS

There are two types of vector multiplications. The product of these two types are known as scalar product and vector product.

If the product of two vectors results in a scalar quantity, it is called scalar product while if the product of two vectors results in a vector quantity, it is called vector product.

Scalar or Dot Product

The scalar product of two vectors \mathbf{A} and \mathbf{B} is written as $\mathbf{A} \cdot \mathbf{B}$ and is defined as:

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta \dots \dots \dots (2.6)$$

where A and B are the magnitudes of vectors \mathbf{A} and \mathbf{B} and θ is the angle between them.

For physical interpretation of dot product of two vectors \mathbf{A} and \mathbf{B} , these are first brought to a common origin (Fig.2.2-a) then, $\mathbf{A} \cdot \mathbf{B} = A(\text{projection of } \mathbf{B} \text{ on } \mathbf{A})$

or

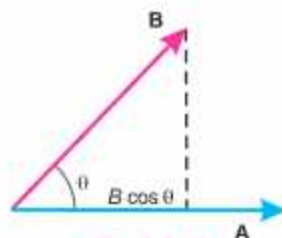


Fig. 2.2 (a)

$\mathbf{A} \cdot \mathbf{B} = A$ (magnitude of component of \mathbf{B} in the direction of \mathbf{A}), Fig.2.2(b)

$$= A (B \cos \theta) = AB \cos \theta$$

Similarly $\mathbf{B} \cdot \mathbf{A} = B (A \cos \theta) = BA \cos \theta$

This type of product when we consider the work done by a force \mathbf{F} whose point of application moves a distance d in a direction making an angle θ with the line of action of \mathbf{F} , as shown in Fig. 2.3.

Work done = (Effective component of force in the direction of motion) x Distance moved

$$= (F \cos \theta) d = Fd \cos \theta$$

Using vector notation:

$$\mathbf{F} \cdot \mathbf{d} = Fd \cos \theta = \text{Work done}$$

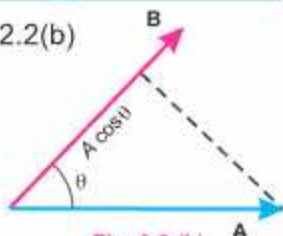


Fig. 2.2 (b)

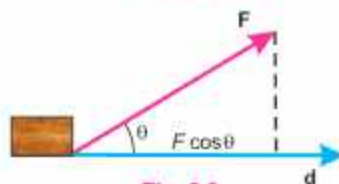


Fig. 2.3

Characteristics of Scalar Product

1. Since $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$ and $\mathbf{B} \cdot \mathbf{A} = BA \cos \theta = AB \cos \theta$, hence, $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$. The order of multiplication is irrelevant. In other words, scalar product is commutative.
2. The scalar product of two mutually perpendicular vectors ($\theta = 90^\circ$) is zero.

$$\mathbf{A} \cdot \mathbf{B} = AB \cos 90^\circ = 0$$

3. The scalar product of two parallel vectors is equal to the product of their magnitudes. Thus, for parallel vectors ($\theta = 0^\circ$).

$$\mathbf{A} \cdot \mathbf{B} = AB \cos 0^\circ = AB$$

For antiparallel vectors ($\theta = 180^\circ$)

$$\mathbf{A} \cdot \mathbf{B} = AB \cos 180^\circ = -AB$$

4. The self product of a vector \mathbf{A} is equal to square of its magnitude.

$$\mathbf{A} \cdot \mathbf{A} = AA \cos 0^\circ = A^2$$

5. Scalar product of two vectors \mathbf{A} and \mathbf{B} in terms of their rectangular components

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \dots\dots\dots(2.7)$$

Equation (2.6) can be used to find the angle between two vectors. Since,

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

Therefore
$$\cos \theta = \frac{A_x B_x + A_y B_y + A_z B_z}{AB} \dots\dots\dots(2.8)$$

Vector or Cross Product

The vector product of two vectors **A** and **B**, is a vector which is defined as:

$$\mathbf{A} \times \mathbf{B} = AB \sin \theta \hat{\mathbf{n}} \dots\dots\dots(2.9)$$

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane containing **A** and **B** as shown in Fig. 2.4 (a). Its direction can be determined by right hand rule. For that purpose, place together the tail of vectors **A** and **B** to define the plane of vectors **A** and **B**. The direction of the product vector is perpendicular to this plane. Rotate the First vector **A** into **B** through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector will be along the erect thumb, as shown in the Fig 2.4 (b). Because of this direction rule, $\mathbf{B} \times \mathbf{A}$ is a vector opposite in sign to $\mathbf{A} \times \mathbf{B}$ (Fig. 2.4-c). Hence,

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \dots\dots\dots (2.10)$$

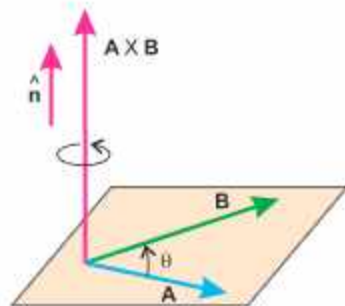


Fig. 2.4(a)

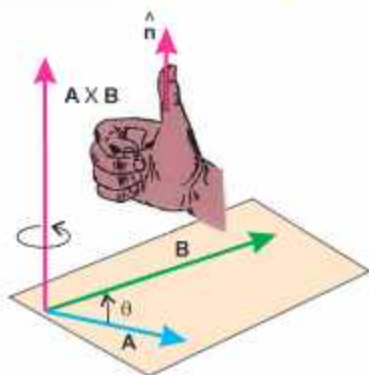


Fig. 2.4(b)

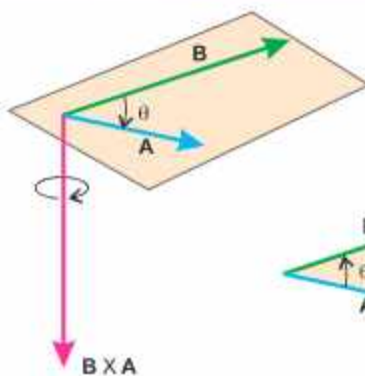


Fig. 2.4(c)

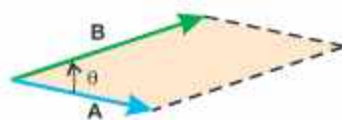


Fig. 2.4(d)

Characteristics of Cross Product

1. Since $\mathbf{A} \times \mathbf{B}$ is not the same as $\mathbf{B} \times \mathbf{A}$, the cross product is non commutative. so,

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

2. The cross product of two perpendicular vectors ($\theta = 90^\circ$) has maximum magnitude.

$$\mathbf{A} \times \mathbf{B} = AB \sin 90^\circ \hat{\mathbf{n}} = AB \hat{\mathbf{n}}$$

3. The cross product of two parallel or anti-parallel vectors is a null vector, because for such vectors $\theta = 0^\circ$ or 180° . Hence,

$$\mathbf{A} \times \mathbf{B} = AB \sin 0^\circ \hat{\mathbf{n}} = 0 \quad \text{or} \quad \mathbf{A} \times \mathbf{B} = AB \sin 180^\circ \hat{\mathbf{n}} = 0$$

As a consequence $\mathbf{A} \times \mathbf{A} = \mathbf{0}$ ($\theta = 0^\circ$)

4. The magnitude of $\mathbf{A} \times \mathbf{B}$ is equal to the area of the parallelogram formed with **A** and **B**

as two adjacent sides (Fig.2.4-d).

Examples of Vector Product

- i. When a force \mathbf{F} is applied on a rigid body at a point whose position vector is \mathbf{r} from any point on the axis about which the body rotates, then the turning effect of the force called the torque τ is given by the vector product of \mathbf{r} and \mathbf{F} .

$$\tau = \mathbf{r} \times \mathbf{F}$$

- ii. The force \mathbf{F} on a particle of charge q and velocity \mathbf{v} in a magnetic field of strength \mathbf{B} is given by vector product of \mathbf{v} and \mathbf{B} .

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

2.4 EQUATIONS OF MOTIONS

Equations of motion can be used to describe the motion of an object in terms of its three kinematic variables: velocity v , position S and time t . There are three ways to pair these variables up: velocity-time, position-time and velocity-position. In this order, they are called first equation of motion, second equation of motion and third equation of motion, respectively.

These equations of motion can only be applied to those objects, which are moving in a straight line with constant acceleration.

Derivation of First Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i . Suppose its velocity changes from initial value v_i to a final value v_f in time interval t . Then the acceleration produced in the body during this time interval is given as:

$$a = \frac{v_f - v_i}{t}$$

Rearranging, we can write

$$v_f - v_i = at$$

$$v_f = v_i + at \dots\dots\dots(2.11)$$

This is the first equation of motion. It correlates the final velocity attained by a body with initial velocity and the time interval t , when moving with constant acceleration a .

Derivation of First Equation of Motion By Graphical Method

First equation of motion can be derived using velocity-time graph for an object moving with initial velocity v_i , final velocity v_f and constant acceleration a .

Let the velocity of a body at point A be v_i which changes to v_f at point B in time interval t as shown in Fig.2.5. A perpendicular BD is drawn from point B to x-axis and another perpendicular BE from B on y-axis, such that

$OA = v_i =$ Initial velocity of the body

$OE = DB = v_f =$ Final velocity of the body

From the graph, it can be observed that;

$$DB = DC + CB$$

$$OB = OA + CB \quad (\text{As } OA = CD)$$

Therefore $v_f = CB + v_i \dots\dots\dots(2.12)$

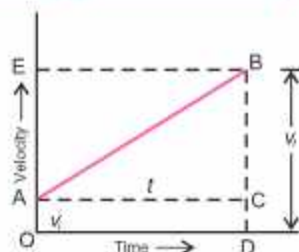


Fig. 2.5: Velocity-Time Graph

The value of CB in the above equation can be determined by taking the slope of line AB , which is equal to acceleration a .

$$a = \frac{CB}{AC}$$

As $AC = t$

So $a = \frac{CB}{t}$

or $BC = at \dots\dots\dots(2.13)$

Combining Eqs. (2.12) and (2.13), we have

$$v_f = v_i + at$$

This is the first equation of motion.

Derivation of Second Equation of Motion

Suppose a body is moving with uniform acceleration a along a straight line with an initial velocity v_i , which becomes v_f after time interval t . Let it cover a distance S in a particular direction during time t , then using the definition of velocity as rate of change of displacement, we can write

$$\text{Velocity} = \text{Displacement} / \text{Time}$$

or $\text{Displacement} = \text{Velocity} \times \text{Time}$

If velocity of the body is not constant, we can use average velocity instead of velocity.

Thus $\text{Displacement} = \text{Average velocity} \times \text{Time}$

$$\text{Displacement} = \frac{(\text{Initial velocity} + (\text{Final velocity}))}{2} \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

Using first equation of motion, $S = \frac{(v_i + v_i + at)}{2} \times t$

$$S = \frac{(2v_i + at)}{2} \times t$$

$$2S = 2v_i t + at^2$$

$$S = v_i t + \frac{1}{2} at^2 \dots\dots\dots(2.14)$$

This is the second equation of motion.

Derivation of Second Equation of Motion by Graphical Method

Second equation of motion can be derived using velocity-time graph for a body moving with initial velocity v_i which attains a final value v_f in time interval t . While moving with constant acceleration a , it covers a displacement S in time t .

It can be seen from the graph that distance travelled by the body is, $S = v \times t$.

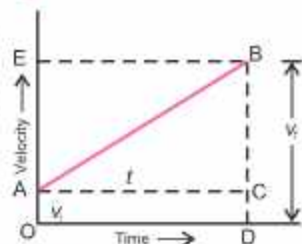


Fig. 2.6: Velocity-Time Graph

Also $S = \text{Area of the figure OACB}$

$$S = (\text{Area of the rectangle OACD}) + (\text{Area of the triangle ABC})$$

$$S = (OA \times OD) + \frac{1}{2} (AC \times BC)$$

As $OA = v_i$ and $OD = AC = t$. So, the above equation becomes:

$$S = v_i \times t + \frac{1}{2} (t \times BC)$$

Here $BC = at$ (From graphical representation of first equation of motion). By putting this value in the above equation, we have

$$S = v_i t + \frac{1}{2} (t \times at)$$

$$S = v_i t + \frac{1}{2} at^2$$

This is the second equation of motion.

Derivation of third equation of motion

Consider a body moving along a straight line with an initial velocity v_i which attains a final value v_f in time t . Let the displacement of the body be S during this time interval. Then, we can write:

$$\text{Displacement} = \left(\frac{\text{Initial velocity} + \text{Final velocity}}{2} \right) \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

$$2S = (v_i + v_f) \times t \dots\dots\dots(2.15)$$

Using the first equation of motion:

$$v_f = v_i + at$$

or $t = \frac{v_f - v_i}{a}$

Putting the value of t in Eq. (2.15)

$$2S = (v_i + v_f) \left(\frac{v_f - v_i}{a} \right)$$

$$2aS = v_f^2 - v_i^2$$

This is the third equation of motion.

Derivation of third Equation of Motion by Graphical method

In the speed-time graph shown in the figure, the total distance S travelled by a body is given by the area OABD under the graph, such that

$$S = \frac{1}{2} (\text{Sum of parallel sides}) \times \text{Height}$$

$$S = \frac{1}{2} (OA + BD) \times OD$$

Since $OA = v_i$, $BD = v_f$ and $OD = t$

The above equation becomes:

$$S = \frac{1}{2} (v_i + v_f) \times t$$

From first equation of motion,

$$t = \frac{v_f - v_i}{a}$$

Putting t in above equation

$$S = \frac{1}{2} (v_i + v_f) \frac{(v_f - v_i)}{a}$$

or

$$S = \frac{1}{2} (v_i + v_f) \frac{(v_f - v_i)}{a}$$

$$2aS = v_f^2 - v_i^2 \dots\dots\dots(2.16)$$

This is the third equation of motion.

The equations of motion are useful in solving the problems relating to linear motion with uniform acceleration, when an object moves along a straight line. If its direction of motion does not change, then all the vector quantities can be manipulated like scalars. In such cases, initial velocity is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity. In the absence of **air resistance**, all objects in free fall at the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration is known as acceleration due to gravity, denoted by **g** and its average value at the Earth surface is taken as 9.8 m s^{-2} in the downward direction. The equation for uniformly accelerated motion can also be applied to free fall motion of the object by replacing **a** by **g**.

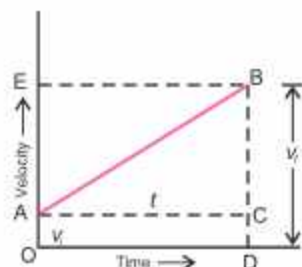


Fig. 2.7: Velocity-Time Graph

Example 2.2 A car travelling at 10 m s^{-1} accelerates uniformly at 2 m s^{-2} . Calculate its velocity after 5 s.

Solution

$$v_i = 10 \text{ m s}^{-1}$$

$$a = 2 \text{ m s}^{-2}$$

$$t = 5 \text{ s}$$

$$v_f = ?$$

Using first equation of motion, we can write

$$v_f = v_i + at$$

$$v_f = 10 \text{ m s}^{-1} + 2 \text{ m s}^{-2} \times 5 \text{ s}$$

$$v_f = 10 \text{ m s}^{-1} + 10 \text{ m s}^{-1}$$

$$v_f = 20 \text{ m s}^{-1}$$

Example 2.3 A car travels with initial velocity of 15 m s^{-1} . It accelerates at a rate of 2 m s^{-2} for 4 seconds. Find the displacement of the car.

Solution

$$v_i = 15 \text{ m s}^{-1}$$

$$a = 2 \text{ m s}^{-2}$$

$$t = 4 \text{ s}$$

Displacement $S = ?$

By using 2nd equation of motion

$$S = v_i t + \frac{1}{2} a t^2$$

Putting the values

$$S = (15 \text{ m s}^{-1} \times 4 \text{ s}) + \frac{1}{2} (2 \text{ m s}^{-2}) (4 \text{ s})^2$$

$$S = 76 \text{ m}$$

Example 2.4 In a short distance race, a contestant in a car starts from rest and reaches the velocity of 300 km h^{-1} , after covering a distance of 0.45 km at a constant acceleration. Find this constant acceleration.

Solution Initial velocity = $v_i = 0$

Final velocity = $v_f = 300 \text{ km h}^{-1}$

$$v_f = \frac{300 \times 1000}{60 \times 60} \text{ m s}^{-1}$$

$$= 83.33 \text{ m s}^{-1}$$

$$\text{Distance covered} = S = 0.45 \text{ km} = 0.45 \times 1000 \text{ m} = 450 \text{ m}$$

Using third equation of motion, we have

$$v_f^2 - v_i^2 = 2aS$$

$$(83.33 \text{ m s}^{-1})^2 - (0)^2 = 2 \times a \times 450 \text{ m}$$

$$a = \frac{6943.88 \text{ m}^2 \text{ s}^{-2}}{900 \text{ m}}$$

$$a = 7.72 \text{ m s}^{-2}$$

2.5 MOTION UNDER GRAVITY

A body falling freely under the action of gravity is the most familiar example of uniformly accelerated rectilinear motion. According to Galileo, all bodies fall freely (in vacuum) under the acceleration due to gravity, denoted by ' g '. Its experimental value is 9.8 m s^{-2} in SI units. This means that different bodies, when allowed to fall from the same height, strike the ground with the same velocity. As regards the sign of g , it is taken positive for a falling body (when initial velocity is zero) and negative for a body projected vertically upward (when initial velocity is not zero).

The equations of motion for a freely falling body, on putting $a = g$, become

$$v_f = v_i + gt$$

$$S = h = v_i t + \frac{1}{2}gt^2$$

$$v_f^2 - v_i^2 = 2gh$$

Example 2.5 An iron ball of mass 1 kg is dropped from a tower. The ball reaches the ground in 3.34 s. Find: (a) the velocity of the ball on striking the ground, (b) the height of the tower.

Solution Since the ball is falling under the action of gravity, we shall put $a = g$ in equations of motion.

Mass of the ball	$m = 1 \text{ kg}$
Time taken to reach ground	$t = 3.34 \text{ s}$
Initial velocity	$v_i = 0$
Final velocity	$v_f = ?$
Acceleration	$a = g = 9.8 \text{ m s}^{-2}$

(a) Using first equation of motion:

$$v_f = v_i + gt$$

$$v_f = 0 + (9.8 \text{ m s}^{-2})(3.34 \text{ s})$$

$$v_f = 32.7 \text{ m s}^{-1}$$

(b) Using third equation of motion:

$$v_f^2 - v_i^2 = 2gh$$

$$(32.7 \text{ m s}^{-1})^2 - (0)^2 = 2 \times 9.8 \text{ m s}^{-2} \times h$$

$$h = \frac{1069.29 \text{ m}^2 \text{ s}^{-2}}{19.6 \text{ m s}^{-2}}$$

$$h = 54.56 \text{ m}$$

2.6 PROJECTILE MOTION

Uptill now we have been studying the motion of a particle along a straight line i.e., motion in one dimension. Now we consider the motion of a ball, when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downward, until it strikes something such as ground. Suppose that the ball leaves the hand of the thrower at point A (Fig 2.8-a) and that its velocity at that instant is completely

horizontal. Let this velocity be v_x . According to Newton's first law of motion, there will be no acceleration in horizontal direction, unless a horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight is the force of gravity. There is no horizontal force acting on it. So, its horizontal velocity will remain unchanged and will be v_x , until the ball hits the ground. The horizontal motion of ball is simple. The ball moves with constant horizontal velocity component. Hence, horizontal distance x is given by

$$x = v_x \times t \dots \dots \dots (2.17)$$

The vertical motion of the ball is also not complicated. It will accelerate downward under the force of gravity and hence $a = g$. This vertical motion is the same as for a freely falling body. Since initial vertical velocity is zero, hence, vertical distance y , using Eq. 2.14 is given by

$$y = \frac{1}{2}gt^2$$

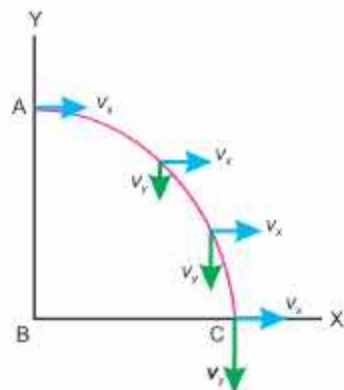


Fig. 2.8(a)

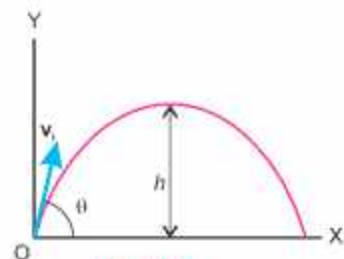


Fig. 2.8(b)

It is not necessary that an object should be thrown with some initial velocity in the horizontal direction. A football kicked off by a player; a ball thrown by a cricketer and a missile fired from a launching pad, all projected at some angles with the horizontal, are called projectiles.

Projectile motion is two dimensional motion under constant acceleration due to gravity.

In such cases, the motion of a projectile can be studied easily by resolving it into horizontal and vertical components which are independent of each other. Suppose that a projectile is fired in a direction angle θ with the horizontal by velocity v_i as shown in Fig.2.8(b). Let components of velocity v_i along the horizontal and vertical directions be $v_i \cos \theta$ and $v_i \sin \theta$, respectively. The horizontal acceleration is $a_x = 0$ because we have neglected air resistance and no other force is acting along this direction, whereas the vertical acceleration is $a_y = g$. Hence, the horizontal component v_x remains constant and at any time t , we have

$$v_{ix} = v_x = v_i \cos \theta \dots\dots\dots (2.18)$$

Now we consider the vertical motion. The initial vertical component of the velocity is $v_i \sin \theta$ in the upward direction.

The vertical component v_y at any instant t can be determined by considering the upward motion of projectile as free fall motion ($a_y = -g$). Using 1st equation of motion:

$$v_{iy} = v_i \sin \theta - gt \dots\dots\dots (2.19)$$

The magnitude of velocity at any instant is:

$$v = \sqrt{v_{ix}^2 + v_{iy}^2} \dots\dots\dots (2.20)$$

The angle ϕ which this resultant velocity makes with the horizontal can be found from

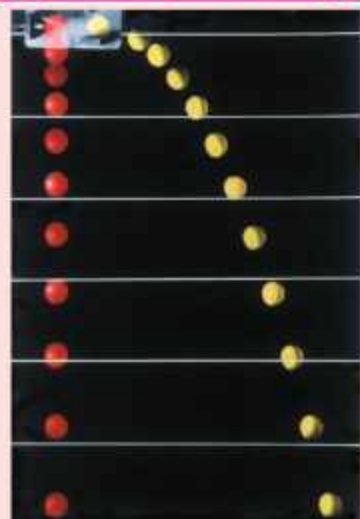
$$\tan \phi = \frac{v_{iy}}{v_{ix}} \dots\dots\dots (2.21)$$

In projectile motion one may wish to determine the height to which the projectile rises, the time of flight and horizontal range. These are described below.

Height of the Projectile

In order to determine the maximum height the projectile attains, we use the equation of motion:

Interesting information



A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

$$2aS = v_f^2 - v_i^2$$

As body moves upward, $a = -g$, the initial vertical velocity $v_y = v_i \sin \theta = v_i$, as $v_y = 0 = v_f$, because the body comes to rest after reaching the highest point. Since

$$S = \text{height} = h$$

$$-2gh = 0 - v_i^2 \sin^2 \theta$$

or
$$h = \frac{v_i^2 \sin^2 \theta}{2g} \dots\dots\dots (2.22)$$

The height of projectile will be reduced in presence of air resistance. In the presence of air resistance, the upward velocity of the projectile will decrease and hence its height will also decrease during time t .

Time of Flight

The time taken by body to cover the distance from the place of its projection to the place where it hits the ground is called the time of flight.

This can be obtained by taking $S = h = 0$, because body goes up and comes back to the same level, thus covering no vertical distance. If the body is projecting with velocity v_i making angle θ with the horizontal, then its vertical component will be $v_i \sin \theta$. Hence, the equation of motion is:

$$S = v_i t + \frac{1}{2} g t^2$$

$$0 = v_i \sin \theta t - \frac{1}{2} g t^2$$

$$t = \frac{2v_i \sin \theta}{g} \dots\dots\dots (2.23)$$

where t is the time of flight of the projectile when it is projected from the ground.

Range of the projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

To determine the range R of the projectile, we multiply the horizontal component of the velocity of projection with total time taken by the body to hit the ground after leaving the point of projection. Thus,

$$R = v_{ix} \times t$$

or
$$R = \frac{v_i \cos \theta \times 2 v_i \sin \theta}{g}$$

or
$$R = \frac{v_i^2}{g} 2 \sin \theta \cos \theta$$

Do you know?

For an angle less than 45° , the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45° , the height attained will be more but the range is again less.

As $2 \sin \theta \cos \theta = \sin 2\theta$, thus, the range of the projectile depends upon the velocity of projection and the angle of projection.

Therefore
$$R = \frac{v_i^2}{g} \sin 2\theta \dots\dots\dots (2.24)$$

For maximum range R , the factor $\sin 2\theta = 1$, so

$$2\theta = \sin^{-1}(1) \text{ or } 2\theta = 90^\circ \text{ or } \theta = 45^\circ$$

Air resistance will slow down projectile forward motion, reducing its velocity v_x . The reduction in v_x will result in a decrease in the range of projectile.

Furthermore, air resistance is not constant throughout the flight of the object. As the object slows down, the air resistance experienced by it also decreases. This means that the object retards more slowly and accelerates more slowly as it falls down. This results in a trajectory that is not perfectly parabolic but is skewed, with steeper descent than ascent.

Example 2.6 A ball is thrown with a speed of 30 m s^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Solution Initially

$$v_x = v_i \cos \theta = 30 \text{ m s}^{-1} \times \cos 30^\circ = 25.98 \text{ m s}^{-1}$$

$$v_y = v_i \sin \theta = 30 \text{ m s}^{-1} \times \sin 30^\circ = 15 \text{ m s}^{-1}$$

As the time of flight, is

$$\begin{aligned} t &= \frac{2v_i \sin \theta}{g} \\ &= \frac{2 \times 30 \text{ m s}^{-1}}{9.8 \text{ m s}^{-2}} \times (0.5) \end{aligned}$$

So
$$t = \frac{2 \times 15 \text{ m s}^{-1}}{9.8 \text{ m s}^{-2}} = 3.1 \text{ s}$$

Height
$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

So
$$h = \frac{(30 \text{ m s}^{-1})^2 (0.5)^2}{19.6 \text{ m s}^{-2}}$$

$$h = 11.5 \text{ m}$$

Range
$$R = \frac{v_i^2}{g} \sin 2\theta = \frac{v_i^2}{g} \sin 60^\circ$$

So
$$\begin{aligned} R &= \frac{(30 \text{ m s}^{-1})^2 \times 0.866}{9.8 \text{ m s}^{-2}} \\ &= 79.53 \text{ m} \end{aligned}$$

For your information



In the presence of air friction the trajectory of a high speed projectile falls short of a parabolic path.

2.7 MOMENTUM

We are aware of the fact that moving object possesses a quality by virtue of which it exerts a force on anything that tries to stop it. The faster the object is travelling, the harder it is to stop it. Similarly, if two objects move with the same velocity, then it is more difficult to stop the massive of the two. Newton referred to this property as "momentum", a vector quantity defined as the product of an object's mass and velocity. This term is now called linear momentum \mathbf{p} of the body and is defined by the relation:

$$\mathbf{p} = m \mathbf{v} \dots\dots\dots(2.25)$$

In this expression, v is the velocity of the mass m . Linear momentum is, therefore, a vector quantity and has the direction of velocity. The SI unit of momentum is kilogram metre per second (kg m s^{-1}). It can also be expressed as newton second (N s).

Momentum and Newton's Second Law of Motion

Consider a body of mass m moving with an initial velocity \mathbf{v}_i . Suppose an external force \mathbf{F} acts upon it for time t after which velocity becomes \mathbf{v}_f . The acceleration \mathbf{a} produced by this force is given by

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, the acceleration is given as:

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\mathbf{F}}{m} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

$$\text{or} \quad \mathbf{F} \times t = m\mathbf{v}_f - m\mathbf{v}_i \dots\dots\dots(2.26)$$

where $m\mathbf{v}_i$ is the initial momentum and $m\mathbf{v}_f$ is the final momentum of the body.

The equation (2.26) shows that change in momentum is equal to the product of force and the time for which force is applied. This form of the second law is more general than the form $\mathbf{F} = m\mathbf{a}$, because it can easily be extended to account for changes as the body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Eq. (2.26)

$$\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{t}$$

Thus, second law of motion can also be stated in terms of momentum as:

Point to ponder!



Which hurt you in the above situations (a) or (b) and think why?

Point to ponder!

Can a moving object experience impulse?

Do you know?

Your hair acts like a crumple zone on your skull. A force of 5 N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Time rate of change of momentum of a body is equal to the applied force.

Impulse

Sometimes we wish to apply the concept of momentum to cases where the applied force is not constant, it acts for very short time. For example, when a bat hits a cricket ball, the force certainly varies from instant to instant during the collision. In such cases, it is more convenient to deal with the product of force and time ($F \times t$) instead of either quantity alone. The product of average force F that acts during time t is called impulse given by

$$\text{Impulse} = F \times t = mv_f - mv_i \dots\dots\dots (2.27)$$

Example 2.7 A 1500 kg car has its velocity reduced from 20 m s^{-1} to 15 m s^{-1} in 3.0 s. How large was the average retarding force?

Solution Using the Eq. (2.27)

$$F \times t = mv_f - mv_i$$

$$F \times 3.0 \text{ s} = 1500 \text{ kg} \times 15 \text{ m s}^{-1} - 1500 \text{ kg} \times 20 \text{ m s}^{-1}$$

$$\begin{aligned} \text{or } F &= -2500 \text{ kg m s}^{-2} \\ &= -2500 \text{ N} = -2.5 \text{ kN} \end{aligned}$$

The negative sign indicates that the force is retarding one.

Law of Conservation of Momentum

Let us consider an isolated system. It is a system on which no external agency exerts any force. For example, the molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion, but being enclosed by glass vessel, no external agency can exert a force on them.

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2 , moving along the same straight line, in the same direction, with velocities v_1 and v_2 respectively. Both the balls collide and after collision, ball of mass m_1 moves with velocity v'_1 and m_2 moves with velocity v'_2 in the same direction as shown in Fig. (2.9).

To find the change in momentum of mass m_1 ,

Using Eq. (2.27) as:

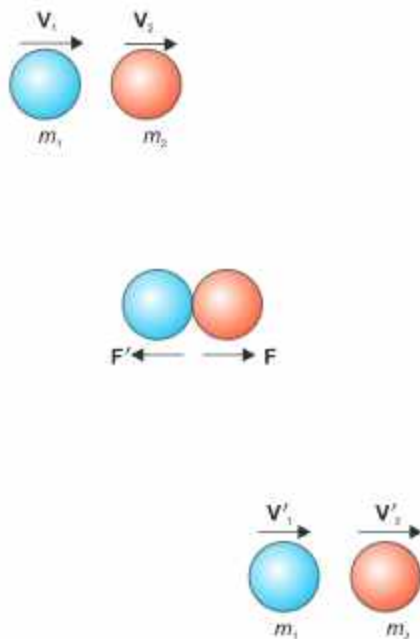


Fig. 2.9

$$\mathbf{F} \times t = m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1$$

Similarly, for the ball of mass m_2 , we have

$$\mathbf{F}' \times t = m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2$$

Adding these two expressions, we have

$$(\mathbf{F} + \mathbf{F}')t = (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2)$$

Since the action force \mathbf{F} is equal and opposite to the reaction force \mathbf{F}' , we have $\mathbf{F}' = -\mathbf{F}$, or $\mathbf{F} + \mathbf{F}' = 0$ so the left hand side of the equation is zero. Hence,

$$0 = (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2)$$

In other words, change in momentum of 1st ball + change in momentum of the 2nd ball is zero.

$$\text{or } (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) = (m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2) \dots\dots\dots(2.28)$$

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

For such a group of objects, if one object within the group experiences a force, there must exist an equal but opposite reaction force on other object in the same group. As a result, the change in momentum of the group of objects as a whole is always zero. This can be expressed in the form of law of conservation of momentum, which states that:

The total linear momentum of an isolated system remains constant.

In applying the conservation law, we must notice that the momentum of a body is a vector quantity.

Example 2.8 Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 m s^{-1} and 4 m s^{-1} , respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 m s^{-1} ?

Solution As both the balls are moving towards one another, so their velocities are of opposite sign. Let us suppose that the direction of motion of 2 kg ball is positive and that of the 3 kg is negative.

The momentum of the system before collision is:

$$\begin{aligned} m_1 v_1 + m_2 v_2 &= 2 \text{ kg} \times 6 \text{ m s}^{-1} + 3 \text{ kg} \times (-4 \text{ m s}^{-1}) \\ &= 12 \text{ kg m s}^{-1} - 12 \text{ kg m s}^{-1} = 0 \end{aligned}$$

Do you know?



When a moving car stops quickly, the passengers move forward towards the windshield. Seatbelts change the forces of motion and prevent the passengers from moving. Thus, the chance of injury is greatly reduced.

Do you know?



A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

$$\begin{aligned}\text{Momentum of the system after collision} &= m_1 v_1' + m_2 v_2' \\ &= 2 \text{ kg} \times v_1' + 3 \text{ kg} \times (-3) \text{ m s}^{-1}\end{aligned}$$

From the law of conservation of momentum

$$\begin{aligned}\left[\begin{array}{l} \text{Momentum of the system} \\ \text{before collision} \end{array} \right] &= \left[\begin{array}{l} \text{Momentum of the system} \\ \text{after collision} \end{array} \right] \\ 0 &= 2 \text{ kg} \times v_1' - 9 \text{ kg m s}^{-1} \\ v_1' &= 4.5 \text{ m s}^{-1}\end{aligned}$$

2.8 ELASTIC AND INELASTIC COLLISIONS

When a tennis ball is dropped on the floor vertically, it may not rebound to its initial height. It is because, a portion of *K.E.* is lost, partly due to friction as the molecules in the ball move past one another when the ball distorts and partly due to its change into heat and sound energies. Similar is the case when two tennis balls collide with certain velocities, their final kinetic energy may be less than the total initial kinetic energy.

A collision in which the *K.E.* of the system is not conserved, is called inelastic collision.

Under certain special conditions, no kinetic energy is lost in the collision or impact on hitting the floor. Such type of collision is said to be elastic collision.

For example, when a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor. It is to be noted that momentum and total energy are conserved in all types of collisions. However, the *K.E.* is conserved only if it is an elastic collision.

Elastic Collisions in One Dimension

Consider two smooth, non-rotating balls of masses m_1 and m_2 moving initially with velocities \mathbf{v}_1 and \mathbf{v}_2 respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after the collision be \mathbf{v}_1' and \mathbf{v}_2' respectively, as shown in Fig.(2.10).

We take the positive direction of the velocity and momentum to the right. By applying the law of conservation of momentum we have

$$(m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) = (m_1 \mathbf{v}_1' + m_2 \mathbf{v}_2')$$

$$m_1(\mathbf{v}_1 - \mathbf{v}_1') = m_2(\mathbf{v}_2' - \mathbf{v}_2) \dots\dots\dots (2.29)$$

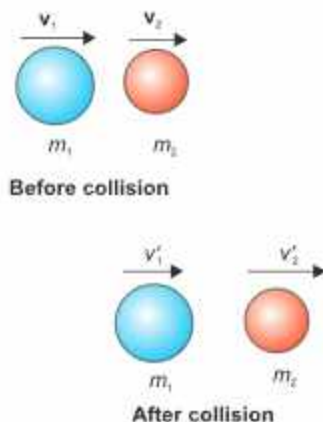


Fig. 2.10

As the collision is elastic, so the *K.E.* is conserved. From the conservation of *K.E.*,

we have
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

or
$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

or
$$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2) \dots\dots\dots(2.30)$$

Dividing Eq. (2.29) by (2.30)

$$(v_1 + v_1') = (v_2' + v_2) \dots\dots\dots(2.31)$$

or
$$(v_1 - v_2) = (v_2' - v_1') = -(v_1' - v_2')$$

We note that, before collision $(v_1 - v_2)$ is the velocity of first ball relative to the second ball. Similarly $(v_1' - v_2')$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

In equations (2.29) and (2.30) m_1, m_2, v_1 and v_2 are known quantities. We solve these equations to find the values of v_1' and v_2' which are unknown. The results are

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \dots\dots\dots(2.32)$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2 \dots\dots\dots(2.33)$$

There are some cases of special interest, which are discussed below:

(i) When $m_1 = m_2$

From Eq. (2.32) and (2.33), we find that

$$v_1' = v_2$$

and
$$v_2' = v_1$$
 (as shown in Fig. 2.11)

(ii) When $m_1 = m_2$ and $v_2 = 0$

In this case, the mass m_2 be at rest, and $v_2 = 0$, then Eqs. (2.32) and (2.33) give

$$v_1' = 0 ; v_2' = v_1$$

When $m_1 = m_2$, then ball of mass m_1 after collision will come to a stop and m_2 will take off with the velocity that m_1 originally had, as shown in Fig.(2.12). Thus when a billiard ball m_1 , moving on

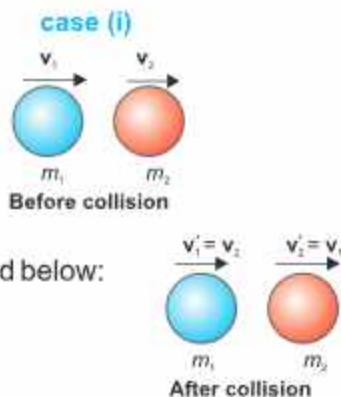


Fig. 2.11

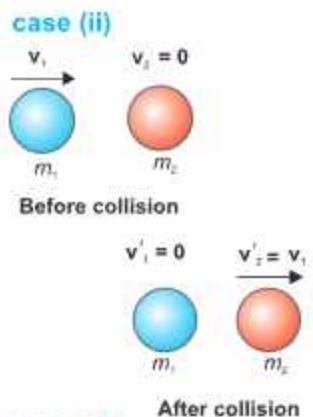


Fig. 2.12

a table collides with exactly similar ball m_2 at rest, the ball m_1 stops while m_2 begins to move with the same velocity, with which m_1 was moving initially.

(iii) When a light body collides with a massive body at rest

In this case initial velocity $v_2 = 0$ and $m_2 \gg m_1$. Under these conditions m_1 can be neglected as compared to m_2 . From Eq. (2.33) and (2.32), we have $v_1' = -v_1$ and $v_2' = 0$.

The result is shown in Fig. (2.13). This means that m_1 will bounce back with the same velocity while m_2 will remain stationary. This fact is used of by the squash player.

(iv) When a massive body collides with light stationary body

In this case, $m_1 \gg m_2$ and $v_2 = 0$, so m_2 can be neglected in Eqs. (2.32) and (2.33). This gives $v_1' \approx v_1$ and $v_2' \approx 2v_1$. Thus, after the collision, there is practically no change in the velocity of massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body, as shown in Fig. (2.14).

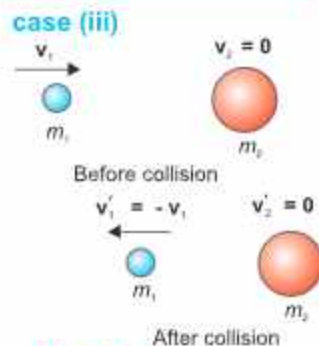


Fig. 2.13

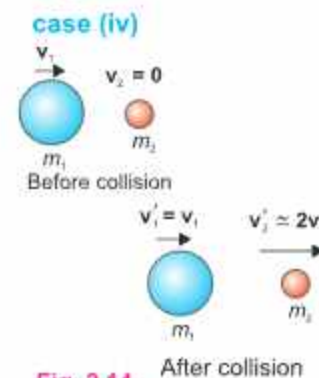


Fig. 2.14

2.9 INELASTIC COLLISION IN ONE DIMENSION

Consider two bodies having masses m_1 and m_2 , moving with velocities v_1 and v_2 , along the same line such that $v_1 > v_2$. In such a case m_1 is regarded as projectile and m_2 as target. After time t both the bodies make inelastic collision and stick together. Let their combined mass become $m_1 + m_2$ which moves with final velocity v_f after collision.

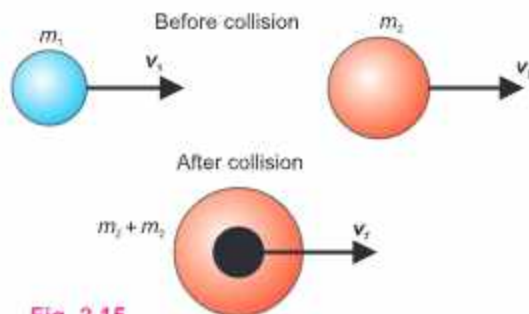


Fig. 2.15

Since the collision is perfectly inelastic, the total momentum of balls is conserved. Using law of conservation of momentum.

Total momentum of system before collision = Total momentum of the system after collision

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1}{m_1 + m_2} v_1 + \frac{m_2}{m_1 + m_2} v_2 \quad \dots\dots\dots(2.34)$$

Which gives the common velocity of the body after inelastic collision.

In a special case when the target m_2 is at rest, $v_2 = 0$, the above equation becomes:

$$\mathbf{v}_f = \frac{m_1}{m_1 + m_2} \mathbf{v}_1$$

It shows that velocity of m_1 is reduced by the mass ratio i.e., $\frac{m_1}{m_1 + m_2}$.

Example 2.9

A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 m s^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic. What would be the velocity of the two balls after the collision?

Solution

$$\begin{array}{lll} m_1 = 70 \text{ g} & v_1 = 9 \text{ m s}^{-1} & v_2 = 0 \\ m_2 = 140 \text{ g} & v'_1 = ? & v'_2 = ? \end{array}$$

We know that;

$$\begin{aligned} v'_1 &= \frac{m_1 - m_2}{m_1 + m_2} v_1 \\ &= \left(\frac{70 \text{ g} - 140 \text{ g}}{70 \text{ g} + 140 \text{ g}} \right) \times 9 \text{ m s}^{-1} = -3 \text{ m s}^{-1} \\ v'_2 &= \frac{2m_1}{m_1 + m_2} v_1 \\ &= \left(\frac{2 \times 70 \text{ g}}{70 \text{ g} + 140 \text{ g}} \right) \times 9 \text{ m s}^{-1} \\ &= 6 \text{ m s}^{-1} \end{aligned}$$

2.10 ELASTIC COLLISION IN TWO DIMENSIONS

Consider the motion of two balls of mass m_1 and m_2 in a straight line with velocities \mathbf{v}_1 and \mathbf{v}_2 respectively undergoing an elastic collision with each other as shown in Fig. 2.16

Assume the bodies move off in different directions after collision with velocities \mathbf{v}'_1 and \mathbf{v}'_2 making angles θ_1 and θ_2 respectively with x-axis.

As, the collision is elastic, so we apply both the laws of conservation of momentum and law of conservation of kinetic energy. Momentum is a vector quantity, we resolve it into its rectangular components and apply the law of conservation of momentum along both axes.

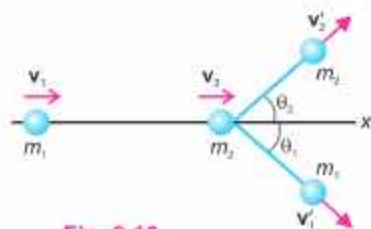


Fig. 2.16

Momentum conservation along x-axis is:

Momentum before collision = momentum after collision

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 \cos \theta_1 + m_2 \mathbf{v}'_2 \cos \theta_2 \dots \dots \dots (2.35)$$

Momentum conservation along y-axis:

Momentum before collision = Momentum after collision

$$0 = m_1 \mathbf{v}'_1 \sin \theta_1 - m_2 \mathbf{v}'_2 \sin \theta_2 \dots \dots \dots (2.36)$$

Conservation of Energy

Kinetic Energy before collision = Kinetic Energy after collision

$$\frac{1}{2} m_1 \mathbf{v}_1^2 + \frac{1}{2} m_2 \mathbf{v}_2^2 = \frac{1}{2} m_1 \mathbf{v}'_1^2 + \frac{1}{2} m_2 \mathbf{v}'_2^2 \dots \dots \dots (2.37)$$

These equations (2.35 to 2.37) are useful in solving problems about elastic collisions in two dimensions.

2.11 INELASTIC COLLISION IN TWO DIMENSIONS

The macroscopic collisions are generally inelastic and do not conserve Kinetic energy.

The perfect inelastic collision is one in which the colliding objects stick together to make a single mass after collision. Its analysis can be carried out as follows:

Let us take two balls having masses m_1 and m_2 , moving with velocities, \mathbf{v}_1 and \mathbf{v}_2 , respectively, in a two-dimensional xy-plane. Assume that the first body is moving along the x-axis while the second body moves in a direction, making an angle θ with x-axis. Both the bodies collide at the origin as shown in the figure 2.17.

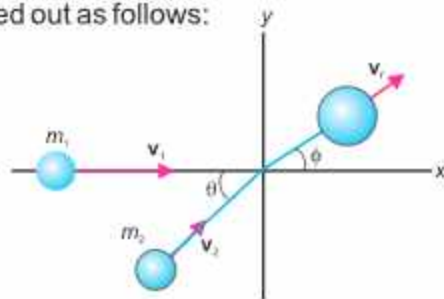


Fig. 2.17

After collision, bodies stick together, having combined mass $M = m_1 + m_2$, which moves with velocity v_f , making an angle ϕ with x-axis.

Momentum in the X-direction:

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 \cos \theta = M \mathbf{v}_f \cos \phi \dots \dots \dots (2.38)$$

Momentum in the y-direction:

$$0 + m_2 \mathbf{v}_2 \sin \theta = M \mathbf{v}_f \sin \phi \dots \dots \dots (2.39)$$

Equation 2.38 and 2.39 can be used to find the final velocity.

Kinetic Energy

Since collision is inelastic, the kinetic energy of colliding system is not conserved. The loss of kinetic energy is computed as follows:

Initial Kinetic Energy

The total initial kinetic energy $K.E.$ of the system before the collision is:

$$(K.E.)_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \dots\dots\dots(2.40)$$

Since $K.E.$ is a scalar quantity, so velocities involving in the formula of $K.E.$ does not require to break velocities into their components.

Final Kinetic Energy

The total final kinetic energy $K.E.$ after the collision (when the objects stick together) is:

$$(K.E.)_f = \frac{1}{2}Mv_f^2 \dots\dots\dots(2.41)$$

where v_f is magnitude of the final velocity which can be calculated from Eq. (2.41).

Energy Loss in the Collision

Since the collision is inelastic, there is a loss in kinetic energy, represented by $\Delta K.E.$

$$\Delta K.E. = (K.E.)_i - (K.E.)_f$$

This lost kinetic energy is transformed into other forms of energy, such as heat, sound, or in deformation.

Some examples of an inelastic collision:

- (i) When a karate chop breaks a pile of bricks, it's an example of an inelastic collision. In this type of collision, the objects involved don't bounce back after impact. Instead, some of the energy from the strike is absorbed by the bricks, converting into heat, sound, and the force needed to break them. This means the energy goes into breaking the bricks rather than causing the hand to rebound. If the Karate chop is not perfectly vertical and involves some horizontal motion, the momentum transfer and the resulting forces will have both horizontal and vertical components.
- (ii) In a car crash, the collision is an inelastic nature. When the vehicles collide and absorb the impact energy, causing them to crumple and deform. This energy absorption slows down the cars, stopping them from bouncing back. Most of the kinetic energy is lost, turning into heat, sound, and damage to the vehicles.
- (iii) In real-world collisions, a ball and bat show an inelastic behaviour. When the bat hits the ball, some of the kinetic energy is lost because the ball deforms, and energy is also converted into heat and sound. Even though the bat is rigid, it does not transfer energy perfectly and absorbs some energy itself. The ball compresses upon impact, which leads to further energy loss. Consequently, not all of the initial kinetic energy is conserved, making the collision overall an inelastic.

2.12 ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines (Fig. 2.18). The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction. The rocket engines continue to expel gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So, instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore work at great heights where very little or no air is present. In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about 10000 kg s^{-1} of fuel and ejects the burnt gases at speeds of over 4000 m s^{-1} . In fact, more than 80% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving others to carry the space craft further up at ever greater speed.

If m is the mass of the gases ejected per second with velocity v relative to the rocket, the change in momentum per second of the ejecting gases is mv . This equals the thrust produced by the engine on the body of the rocket. So, the acceleration 'a' of the rocket is

$$a = \frac{mv}{M} \dots\dots\dots (2.42)$$

where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

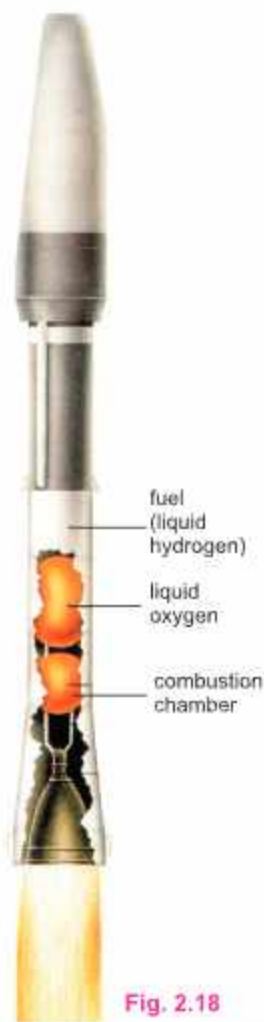


Fig. 2.18

Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals the gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

- 2.1 The angle at which dot product becomes equal to cross product:
(a) 65° (b) 45° (c) 76° (d) 30°
- 2.2 The projectile gains its maximum height at an angle of:
(a) 0° (b) 45° (c) 60° (d) 90°
- 2.3 The scalar product of two vectors is maximum if they are:
(a) perpendicular (b) parallel (c) at 30° (d) at 45°
- 2.4 The range of projectile is same for two angles which are mutually:
(a) perpendicular (b) supplementary
(c) complementary (d) 270°
- 2.5 The acceleration at the top of a trajectory of projectile is:
(a) maximum (b) minimum (c) zero (d) g
- 2.6 SI unit of impulse is:
(a) kg m s^{-2} (b) N m (c) N s (d) N m^2
- 2.7 The rate of change of momentum is:
(a) force (b) impulse (c) acceleration (d) power
- 2.8 As rocket moves upward during its journey, then its acceleration goes on:
(a) increasing (b) decreasing
(c) remains same (d) it moves with uniform velocity
- 2.9 Elastic collision involves:
(a) loss of energy
(b) gain of energy
(c) no gain, no loss of energy
(d) no relation between energy and elastic collision

Short Answer Questions

- 2.1 State right hand rule for two vectors with reference to vector product.
- 2.2 Define impulse and show how it is related to momentum.
- 2.3 Differentiate between an elastic and an inelastic collision.
- 2.4 Show that rate of change in momentum is equal to force applied. Also state Newton's second law of motion in terms of momentum.
- 2.5 State law of conservation of linear momentum. Also state condition under which it holds.

- 2.6 Show that range of projectile is maximum at an angle of 45° .
- 2.7 Find the time of flight of a projectile to reach the maximum height.
- 2.8 The maximum horizontal range of a projectile is 800 m. Find the value of height attained by the projectile at $\theta = 60^\circ$.

Constructed Response Questions

- 2.1 Why does a hunter aiming a bird in a tree miss the target exactly at the bird?
- 2.2 A person falling on a heap of sand does not hurt more as compared to a person falling on a concrete floor. Why?
- 2.3 State the conditions under which birds fly in air.
- 2.4 Describe the circumstances for which velocity and acceleration of a vehicle are:
- \mathbf{v} is zero but \mathbf{a} is not zero
 - \mathbf{a} is zero but \mathbf{v} is not zero
 - perpendicular to one another
- 2.5 Describe briefly effects of air resistance on the range and maximum height of a projectile.

Comprehensive Questions

- 2.1 Define and explain scalar product. Write down its important characteristics.
- 2.2 Define and explain vector product of two vectors. Discuss important characteristics of vector product.
- 2.3 Derive three equations of motion by graphical method.
- 2.4 What is projectile motion? Explain.
- 2.5 Derive the following expressions for projectile motion:
- time of flight
 - height attained
 - range for projectile.
- 2.6 Explain elastic collision in one dimension. Show that magnitude of relative velocities before and after collision are equal.
- 2.7 Explain elastic collision in two dimensions.
- 2.8 Explain an inelastic collision in one and two dimensions.

Numerical Problems

- 2.1 The magnitude of cross and scalar products of two vectors are $4\sqrt{3}$ and 4, respectively. Find the angle between the vectors. (Ans: 60°)
- 2.2 A helicopter is ascending vertically at the rate of 19.6 m s^{-1} . When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground? (Ans: 8.0 s)

- 2.3 If $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$, then prove that \mathbf{A} and \mathbf{B} are perpendicular to each other.
(Ans: $\theta = 90^\circ$)
- 2.4 A body of mass M at rest explodes into 3 pieces, two of which of mass $M/4$ each are thrown off in perpendicular directions with velocities of 3 m s^{-1} and 4 m s^{-1} , respectively. Find the velocity of 3rd piece with which it will be flown away.
(Ans: 2.5 m s^{-1} , opposite to resultant velocity vector of two pieces)
- 2.5 A cricket ball is hit upward with velocity of 20 m s^{-1} at an angle of 45° with the ground. Find its:
(a) time of flight (b) maximum height (c) how far away it hits the ground
(Ans: 2.9 s, 41 m, 10.2 m)
- 2.6 A 20 g ball hits the wall of a squash court with a constant force of 50 N. If the time of impact of force is 0.50 s, find the impulse.
(Ans: 25 N s)
- 2.7 A ball is kicked by a footballer. The average force on the ball is 240 N, and the impact lasts for a time interval of 0.25 s.
(a) Calculate change in momentum
(b) State the direction of change in momentum
(Ans: (a) 60 N s, (b) In the direction of force)
- 2.8 An aeroplane is moving horizontally at a speed of 200 m s^{-1} at a height of 8 km to drop a bomb on a target. Find horizontal distance from the target at which the bomb should be released.
(Ans: 8.08 km)
- 2.9 Why does range R of a projectile remain the same when angle of projection is changed from θ to $\theta' = 90^\circ - \theta$. Also show that for complementary angles of projection, the ratio R/R' is equal to 1.
- 2.10 A trolley of mass 1.0 kg moving with velocity 1.0 m s^{-1} collides with a similar trolley at rest:
(i) after collision, the 1st trolley comes to rest whereas the second starts moving with velocity of 1.0 m s^{-1} in the same direction. Show that it is an example of an elastic collision.
(ii) after the collision, they stick together and move away with a velocity of 0.5 m s^{-1} . Show that it is an example of an inelastic collision.
- 2.11 A railway wagon of mass $4 \times 10^4 \text{ kg}$ moving with velocity of 3 m s^{-1} collides with another wagon of mass $2 \times 10^4 \text{ kg}$ which is at rest. They stick together and move off together. Find their combined velocity.
(Ans: 2 m s^{-1})
- 2.12 A car with mass 575 kg moving at 15.0 m s^{-1} smashes into the rear end of a car with mass 1575 kg moving at 5 m s^{-1} in the same direction.
(a) What is the final velocity if the wrecked car lock together?
(b) How much kinetic energy is lost in the collision?
(Ans: (a) 7.67 m s^{-1} , (b) $2.11 \times 10^4 \text{ J}$)

Learning Objectives

After studying this chapter, the students will be able to:

- ◆ Express angles in radians
- ◆ Define and calculate angular displacement, angular velocity and angular acceleration [This involves use of $S = r\theta$, $v = r\omega$, $\omega = 2\pi r/T$, $a = r\omega^2$, and $a = v^2/r$ to solve problems]
- ◆ Use equations of angular motion to solve problems involving rotational motions.
- ◆ Discuss qualitatively motion in a curved path due to a perpendicular force.
- ◆ Define and calculate centripetal force [Use $F_c = mr\omega^2$, $F_c = mv^2/r$]
- ◆ Analyze situations involving circular motion in terms of centripetal force [e.g. situations in which centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force.]
- ◆ Define and calculate average orbital speed for a satellite. [from the equation $v = 2\pi r/T$ where r is the average radius of the orbit and T is the orbit period; apply this equation to solve numerical problems]
- ◆ Explain why the objects in orbiting satellites appear to be weightless.
- ◆ Describe how artificial gravity is created to counter weightlessness.
- ◆ Define and calculate moments of inertia of a body and angular momentum.
- ◆ Derive and apply the relation between torque, moment of inertia and angular acceleration. Illustrate the applications of conservation of angular momentum in real life. [such as by flywheels to store rotational energy, by gyroscopes in navigation systems, by ice skaters to adjust their angular velocity]
- ◆ Describe how a centrifuge is used to separate materials using centripetal force

Among all possible motions of the material bodies, the circular motion is one that appears to be working in the most of the natural world. Satellites moving in circular orbits around the Earth, orbital and spin motion of the Earth itself, a car turning around a curved road, and a stone whirled around by a string are the familiar examples. When objects move in circular paths, their direction is continuously changing. Since velocity is a vector quantity, this change of direction means that their velocities are not constant. In this chapter, we will study, circular motion, rotational motion, moment of inertia, angular momentum and the related topics.

3.1 ANGULAR MEASUREMENTS

Consider an angle subtended at the centre 'O' of a circle by an arc 'AB' as shown in Fig. 3.1. If the length of the arc 'AB' is equal to the radius ' r ' of the circle, then the angle is called one radian. It is the SI unit of angular measurement and its symbol is "rad".

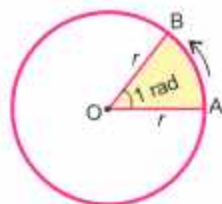


Fig. 3.1

Angular Displacement

Consider the motion of a single particle P of mass m in a circular path of radius r . Suppose this motion is taking place by attaching the particle P at the end of a massless rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. 3.2 (a). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. 3.2 (b). The z-axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t , its position is OP_1 , making angle θ with x-axis. At a later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis (Fig. 3.2-c).

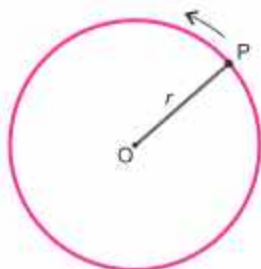


Fig. 3.2(a)

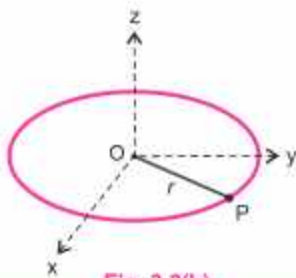


Fig. 3.2(b)

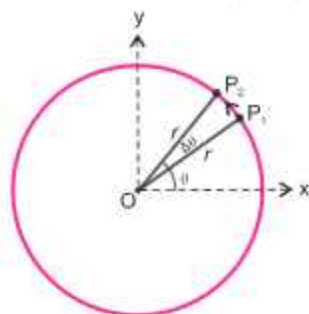


Fig. 3.2(c)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt . For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clock wise.

The direction associated with $\Delta\theta$ is along the axis of rotation and is given by right hand rule as shown in Fig 3.2 (d) which states that:

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement.

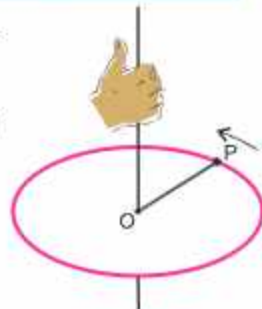


Fig. 3.2(d)

Three units are generally used to express angular displacement, namely degrees, revolution and radian. Consider an arc of length S of a circle of radius r (Fig. 3.3) which subtends an angle θ at the centre of the circle. Its value in radians (rad) is given as:

$$\theta = \frac{S}{r}$$

or $S = r\theta$ (where θ is in radian) (3.1)

If OP is rotating, the point P covers a distance $S = 2\pi r$ in one revolution of P. In radian, it would be:

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi$$

So 1 revolution = 2π rad = 360°

or 1 rad = $\frac{360^\circ}{2\pi} = 57.3^\circ$

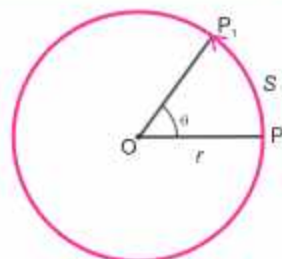


Fig. 3.3

Angular Velocity

Very often, we are interested in knowing how fast or how slow a body is rotating. It is determined by its angular velocity defined as the rate at which the angular displacement is changing with time. Referring to Fig. 3.2(c), if $\Delta\theta$ is the angular displacement during the time interval Δt , the average angular velocity ω_{av} during this interval is given by

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \quad \text{.....} \quad (3.2)$$

The instantaneous angular velocity ω is the limit of the ratio $\Delta\theta/\Delta t$ as Δt , approaches to zero.

Thus
$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \quad \text{.....} \quad (3.3)$$

In the limit when Δt approaches zero, the angular displacement would be infinitesimally small. So, it would be a vector quantity and the angular velocity as defined by Eq. 3.3 would also be a vector. Its direction is along the axis of rotation and is given by right hand rule as described earlier.

Angular velocity is measured in radians per second which is the SI unit. Sometimes it is also given in terms of revolution per minute (rpm).

Angular acceleration

When we switch on an electric fan, we notice that its angular velocity goes on increasing till it becomes uniform. We say that it has an angular acceleration. We define angular acceleration as the rate of change of angular velocity. If ω_1 and ω_2 are the values of instantaneous velocity of a rotating body at instants t_1 and t_2 , the average angular acceleration during the interval $t_2 - t_1$ is given by

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \quad \text{.....} \quad (3.4)$$

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches zero. Therefore, instantaneous angular acceleration α is given by

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} \quad \dots\dots\dots (3.5)$$

The angular acceleration is also a vector quantity whose magnitude is given by Eq. 3.5 and its direction is along the axis of rotation. Angular acceleration is expressed in rad s^{-2} .

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now consider the rotation of a rigid body as shown in Fig. 3.4. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation usually referred as the reference line. As the body rotates, line OP also rotates with the same angular velocity and angular acceleration. Thus, the rotation of a rigid body can be described by the rotation of the reference line OP and all the terms that we defined with the help of rotating line OP are also valid for the rotational motion of a rigid body. Henceforth, while dealing with rotation of a rigid body, we will replace it by its reference line OP.

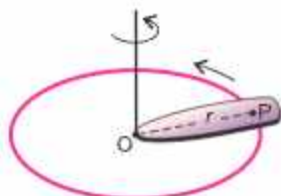


Fig. 3.4

Relation between Angular and Linear Velocities

Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. 3.5 (a).

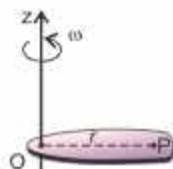


Fig. 3.5(a)

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity v whereas the line OP rotates with angular velocity ω as shown in Fig. 3.5 (b). We are interested in finding out the relation between ω and v . As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, let us consider only its magnitude only which can also be treated as a scalar.

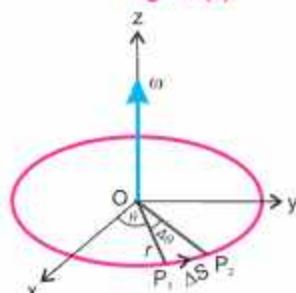


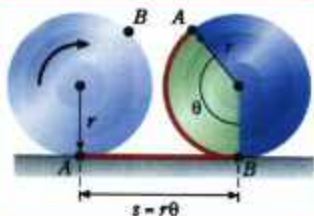
Fig. 3.5(b)

$$P_1P_2 = \Delta S$$

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP covers an angular displacement $\Delta\theta$ radian. So, ΔS and $\Delta\theta$ are related by Eq. 3.1as:

$$\Delta S = r\Delta\theta$$

For your information



As the wheel turns through an angle θ , it lays out a tangential distance $S = r\theta$.

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \quad \dots\dots\dots (3.6)$$

In the limit when linear $\Delta t \rightarrow 0$ the ratio $\Delta S/\Delta t$ represents v , the magnitude of the linear velocity with which point P is moving on the circumference of the circle. Similarly $\Delta \theta/\Delta t$ represents the angular velocity ω of the reference line OP. So, Eq. 3.6 becomes:

$$v = r\omega \quad \dots\dots\dots (3.7)$$



Point to ponder!

You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

From Fig. 3.5(b), it can be seen that the point P is moving along the arc P_1P_2 . In the limit when $\Delta t \rightarrow 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1 . Thus, the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why, the linear velocity of the point P is also known as tangential velocity.

Similarly, Eq. 3.7 shows that if the reference line OP is rotating with an angular acceleration α , the point P will also have a linear or tangential acceleration a_t . Using Eq. 3.7 it can be shown that the two accelerations are related by

$$a_t = r\alpha \quad \dots\dots\dots (3.8)$$

Eqs. 3.7 and 3.8 show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body rotating about a fixed axis do have the same angular displacement, angular speed and angular acceleration at any instant. Thus, by the use of angular variables, we can describe the motion of the entire body in a simple way.

Equations of Angular Motion

The equations (3.2, 3.3, 3.4 and 3.5) of angular motion are exactly analogous to those in linear motion if θ , ω and α be replaced by S , v and a , respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear Equations	Angular Equations
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t \quad \dots\dots\dots (3.9)$
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad \dots\dots\dots (3.10)$
$S = v_i t + \frac{1}{2} at^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2 \quad \dots\dots\dots (3.11)$

The angular equations 3.9 to 3.11 hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence, they can be manipulated as scalars.

Example 3.1 An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Solution In this problem, we have

$$\omega_i = 3.0 \text{ rev s}^{-1}, \quad \omega_f = 0, \quad t = 18.0 \text{ s} \quad \text{and} \quad \alpha = ? , \quad \theta = ?$$

From Eq. 3.9, we have

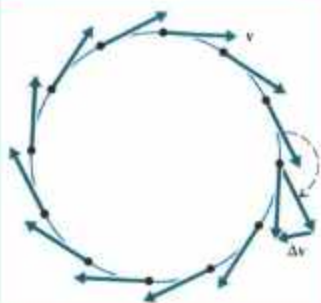
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{(0 - 3.0) \text{ rev s}^{-1}}{18.0 \text{ s}} = -0.167 \text{ rev s}^{-2}$$

and from Eq. 3.11, we have

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\begin{aligned} &= 3.0 \text{ rev s}^{-1} \times 18.0 \text{ s} + \frac{1}{2} (-0.167 \text{ rev s}^{-2}) \times (18.0 \text{ s})^2 \\ &= 27 \text{ rev} \end{aligned}$$

Do you know?



Direction of motion changes continuously in circular motion.

3.2 CENTRIPETAL FORCE

Newton's second law of motion states that when a force acts on a body, it produces acceleration in the same direction. A force acting on a moving body along the direction of its velocity will change magnitude of the velocity (speed) keeping the direction unchanged. On the other hand, a constant force acting perpendicular to the velocity of a body moving in a circular path will change the direction but magnitude of velocity (speed) will remain the same. Such force makes the body move in a circle by producing a radial (or centripetal) acceleration and is called centripetal force (Centre seeking) force. Figure 3.6(a) shows a ball tied at the end of a string is whirled in a horizontal surface. It would not continue in a circular path if the string is snapped. Careful observation shows at once that if the string snaps, when the ball is at the point A, in Fig. 3.6 (b), the ball will follow the straight line path AB which is tangent AB at point A.

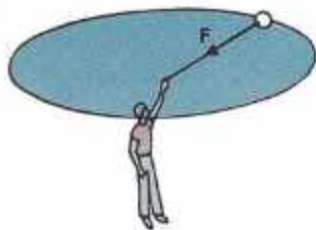


Fig. 3.6(a)

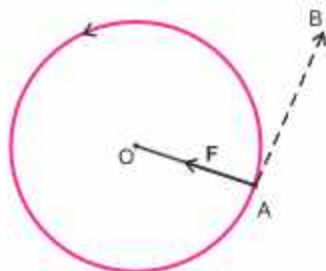


Fig. 3.6(b)

Thus, a force is needed to change the direction of velocity or motion of a body continuously at each point in circular motion moving with uniform speed. The force that does not alter speed but only direction at each point is a perpendicular force which acts along the radius of the circular path. This force always pulls the object towards the centre

of the circular path. Its direction is perpendicular to the tangential velocity at each point.

The force needed to bend the straight path of the particle into a circular path is called the centripetal force.

For a body of mass m moving with velocity v in a circular path of radius r , centripetal force F_c is given by

$$F_c = ma_c = \frac{mv^2}{r} \dots\dots\dots (3.12)$$

where $a = v^2/r$ is the centripetal acceleration and its direction is towards the centre of the circle. As $v = r\omega$, so the above equation becomes:

$$F_c = mr\omega^2 \dots\dots\dots (3.13)$$

Example 3.2 If a CD spins at 210 rpm, what is the radial acceleration of a point on the outer rim of the CD? The CD is 12 cm in diameter.

Solution We convert 210 rpm into a frequency in revolutions per second (Hz).

$$\text{Thus } f = 210 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 3.5 \frac{\text{rev}}{\text{s}} = 3.5 \text{ Hz}$$

For each revolution, the CD rotates through an angle of 2π radians. The angular velocity is:

$$\omega = 2\pi f = 2\pi \text{ rad} \times 3.5 \text{ s}^{-1} = 7.0 \pi \text{ rad s}^{-1}$$

The radial acceleration is:

$$a = \omega^2 r = (7.0\pi \text{ rad s}^{-1})^2 \times 0.06 \text{ m} = 29 \text{ m s}^{-2}$$

Example 3.3 A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 3.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?

Solution For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the string and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

$$T + w = \frac{mv^2}{r} \quad \text{as } w = mg, \text{ therefore,}$$

$$\therefore T = \frac{mv^2}{r} - mg = m \left(\frac{v^2}{r} - g \right)$$

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

Do you know?



Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

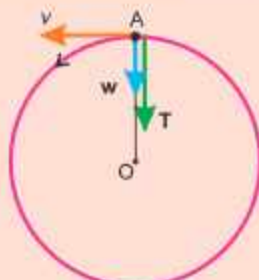


Fig. 3.7

Examples of centripetal force

In every circular or orbital motion, centripetal force is needed which is provided by some agency.

1. When a ball is whirled in a horizontal circle with the help of a string, then tension in the string provides necessary centripetal force.
2. For an object placed on a turntable, the friction is the centripetal force.
3. The gravitational force is the cause of the Earth orbiting around the Sun, Moon and artificial satellites revolving around the Earth.
4. A normal or perpendicular magnetic force compels a charge particle moving along a straight path into a circular path.
5. When a vehicle takes turn on a road, it also needs centripetal force which is provided by the friction between the tyres and the road. If the road is slippery, then at high speed, the friction may not be sufficient enough to provide necessary centripetal force.



Hence, vehicle will not be able to take turn and may skid or may even be toppled. To overcome this difficulty, the highway road is banked on turns. That is, the outer edge of the track is kept slightly higher than that of the inner edge.

Applications of Centripetal Force

We know that an object moves in a circle because of centripetal force. If the magnitude of applied force falls short of required centripetal force then the object will move away from the centre of the circle. The centrifuge (Fig. 3.8-a) functions on this basic principle.

Centrifuge: It is one of the most useful laboratory device. It helps to separate out denser and lighter particles from a mixture. The mixture is rotated at high speed for a specific time. In a laboratory setup, sample tubes are used where the denser particles will settle at the bottom and lighter particles will rise to the top of the sample tubes (Fig. 3.8-b).

The **dryer** of the washing machine also functions on the principle of centrifuge. The dryer consists of a long cylinder with hundreds of small holes on its wall. Wet clothes are piled up in this cylinder, which is then rotated rapidly about its axis. Water moves outward to the walls of the cylinder and thus, drained out through the holes. In this way, clothes become dry quickly.



Fig. 3.8 (a)

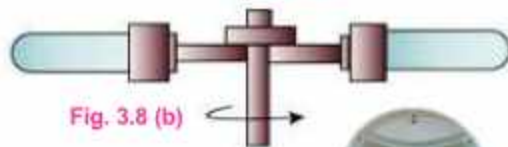


Fig. 3.8 (b)



Fig. 3.9

Cream separator is another practical device which is used to separate cream from the milk. In this machine, milk is whirled rapidly. Since milk is a mixture of light and heavy particles, when it is rotated, the light particles gather near the axis of rotation whereas the heavy particles will go outwards and hence, cream can easily be separated from milk.



Fig. 3.10

3.3 ARTIFICIAL SATELLITES

Satellites are objects that orbit in nearly circular path around the Earth. They are put into orbit by rockets and are held in orbits by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 m s^{-2} towards the centre of the Earth. If there is no gravitational pull, they would fly off in a straight line along tangent to the orbit. When the satellite is moving in a circle, it has an acceleration $\frac{v^2}{R}$. In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity and we have

$$g = \frac{v^2}{R} \quad \dots\dots\dots (3.14)$$

where v is the orbital velocity and R is the radius of the Earth (6400 km). From Eq. 3.14, we have

$$\begin{aligned} v &= \sqrt{gR} \\ &= \sqrt{9.8 \text{ m s}^{-2} \times 6.4 \times 10^6 \text{ m}} \\ &= 7.9 \times 10^3 \text{ m s}^{-1} = 7.9 \text{ km s}^{-1} \end{aligned}$$

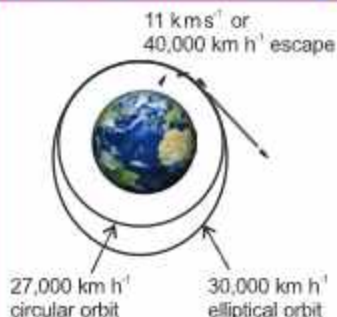
This is the minimum velocity necessary to put a satellite into the orbit, called the critical velocity. The period T is given by

$$\begin{aligned} T &= \frac{2\pi R}{v} = 2 \times 3.14 \times \frac{6400 \text{ km}}{7.9 \text{ km s}^{-1}} \\ &= 5060 \text{ s} = 84 \text{ min approx.} \end{aligned}$$

If, however, a satellite in a circular orbit is at a distance h much greater than R above the Earth's surface, we must take into account the experimental fact that the gravitational acceleration decreases inversely as the square of the distance from the centre of the Earth (Fig. 3.11).

The higher the satellite, the slower will be the required speed and longer it will take to complete one revolution around the Earth.

For your information



Satellites Orbits

Do you know?

Newton had predicted about the artificial satellites 300 years ago. The above figure has been taken from his well-known book "Principia Mathematica". According to this book, if an object is thrown horizontally with a particular speed from a place which is sufficiently high, it will start revolving around the Earth.

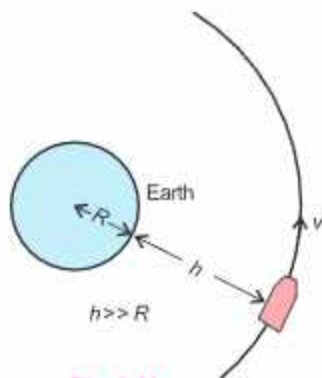


Fig. 3.11

Orbital Velocity

Figure 3.12 shows a satellite going round the Earth in a circular path. Let the mass of the satellite be m_s and v is its orbital speed. The mass of the Earth is M and r represents the radius of the orbit. A centripetal force $m_s v^2/r$ is required to hold the satellite in the orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equating the gravitational force to the required centripetal force, we have

$$\frac{Gm_s M}{r^2} = \frac{m_s v^2}{r}$$

or
$$v = \sqrt{\frac{GM}{r}} \quad \dots\dots\dots (3.15)$$

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus, any satellite orbiting at distance r from the Earth's centre must have the orbital speed given by Eq. 3.15. Any speed less than this will bring the satellite tumbling back to the Earth.

Example 3.4 An Earth satellite is in circular orbit at distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Solution

As $r = R + h = (6400 + 384000)$ km = 390400 km

$$\begin{aligned} \text{Using } v &= \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 6 \times 10^{24} \text{ kg}}{390400 \text{ km}}} \\ &= 1.025 \text{ km s}^{-1} \end{aligned}$$

Also

$$\begin{aligned} T &= \frac{2\pi R}{v} = 2 \times 3.14 \times 390400 \text{ km} \times \frac{1}{1.025 \text{ km s}^{-1}} \times \frac{1 \text{ day}}{60 \times 60 \times 24 \text{ s}} \\ &= 27.7 \text{ days} \end{aligned}$$

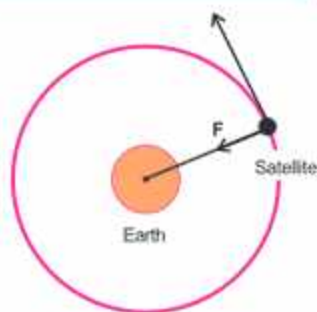


Fig. 3.12

Tidbits

The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

Tidbits



In 1984, at a height of 100 km above Hawaii island with a speed of 29000 km h⁻¹ Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Do you know?

Your weight slightly changes when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

Weightlessness in Satellites

When a satellite is launched by a rocket in its desired orbit around the Earth, then it has been observed practically that everything inside the satellite experiences weightlessness because the satellite is accelerating towards the centre of the Earth as a freely falling body.

Consider a satellite of mass M revolving in its orbit of radius r around the Earth. A body of mass m inside the satellite suspended by a spring balance from the ceiling of the satellite is under the action of two forces. That is, its weight mg acting downward, while the supporting force, called normal force F_N or tension in the spring acting upward, as shown in Fig. 3.13. Their resultant force is equal to the centripetal force required by the mass m which is acting towards the centre of the Earth, and is expressed as:

$$F_c = mg - F_N \quad \dots\dots (3.16)$$

where $F_c = \frac{mv^2}{r}$

$$\text{Hence } \frac{mv^2}{r} = mg - F_N \quad \dots\dots (3.16-a)$$

It may be noted that the centripetal force responsible for the revolution of the satellite of mass M around the Earth is provided by the gravitational force of attraction between the Earth and the satellite.

$$F_g = F_c$$

$$Mg = \frac{Mv^2}{r}$$

$$g = \frac{v^2}{r}$$

Hence, Eq. 3.16(a) becomes:

$$mg = mg - F_N$$

or $F_N = 0$

This shows that the supporting force which is acting on a body inside the satellite is zero. Therefore, the bodies as well as the astronauts in a satellite find themselves in a state of apparent weightlessness.

Artificial Gravity

In a gravity free space, there will be no force that will push anybody to any side of the spacecraft. If this spacecraft is to stay in the orbit over an extended period of time, the weightlessness may affect the performance of the astronauts present in that spacecraft. To overcome this difficulty, an artificial gravity can be created in the spacecraft. This could enable the crew of the space ships to function in an almost normal manner. For this situation to prevail, the spaceship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.

Consider a spacecraft of the shape as shown in Fig. 3.14. The outer radius of the

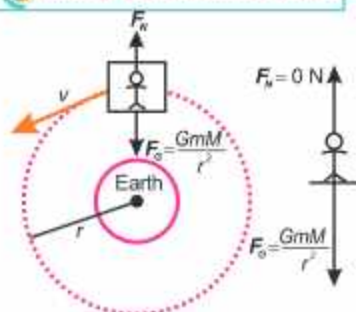


Fig. 3.13

Can you tell?

When a bucket full of water is rapidly whirled in a vertical circular path, water does not fall out even if the bucket is inverted at the maximum height. Why is it so?



Astronaut floating inside the cabin of a spaceship.

spaceship is R and it rotates around its own central axis with angular speed ω , then its angular acceleration a_c is

$$a_c = R \omega^2$$

But $\omega = \frac{2\pi}{t}$ where t is the period of revolution of spaceship

Hence
$$a_c = R \frac{(2\pi)^2}{t^2} = R \frac{4\pi^2}{t^2}$$

As frequency $f = 1/t$, therefore,

$$a_c = R 4\pi^2 f^2$$

or
$$f^2 = \frac{a_c}{4\pi^2 R} \quad \text{or} \quad f = \frac{1}{2\pi} \sqrt{\frac{a_c}{R}}$$

As described above, the force of gravity provides the required centripetal acceleration, therefore,

$$a_c = g$$

So
$$f = \frac{1}{2\pi} \sqrt{\frac{g}{R}} \quad \dots\dots\dots (3.17)$$

When the spaceship rotates with this frequency, the artificial gravity like the Earth is provided to the inhabitants of the spaceship.

3.4 MOMENT OF INERTIA

Consider a mass m attached to the end of a massless rod as shown in Fig. 3.15. Assume that the bearing at the pivot point O is frictionless. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence, this will accelerate the mass according to:

$$F = ma$$

In doing so, the force will cause the mass to rotate about O . Since tangential acceleration a_T is related to angular acceleration α by the equation,

$$a_T = r\alpha$$

So
$$F = mr\alpha$$

As turning effect is produced by torque τ , it would, therefore, be better to write the equation for rotation in terms of torque. This can be done by multiplying both sides of the above equation by r . Thus,

$$rF = \tau = \text{torque} = mr^2\alpha$$

which is rotational analogue of the Newton's second law of motion, $F = ma$.

Here F is replaced by τ , a by α and m by mr^2 . The quantity mr^2 is known as the moment

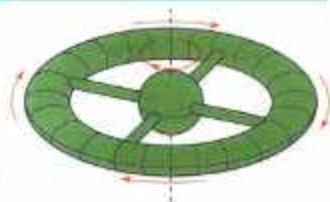


Fig. 3.14

Do you know?



The surface of the rotating space-ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

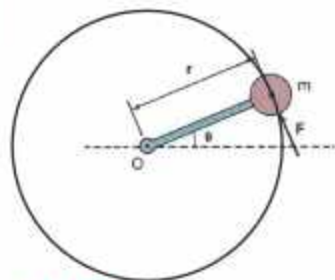


Fig. 3.15

The force F causes a torque about the axis O and gives the mass m an angular acceleration α about the pivot point.

of inertia and is represented by I . The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2 .

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means the mass distribution is not uniform. As shown in Fig. 3.16(a), the rigid body is made up of n small pieces of masses m_1, m_2, \dots at distances r_1, r_2, \dots from the axis of rotation O . Let the body be rotating with the angular acceleration α , so the magnitude of the torque acting on m_1 is

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is

$$\tau_2 = m_2 r_2^2 \alpha_2$$

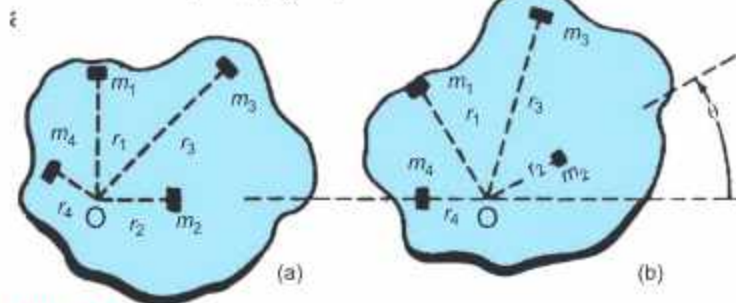


Fig. 3.16

Each small piece of mass within a large, rigid body undergoes the same angular acceleration about the pivot point.

Since the body is rigid, so all the masses are rotating with the same angular acceleration α .

Total torque τ_{total} is then given by

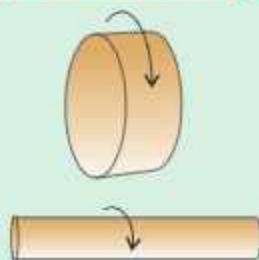
$$\begin{aligned} \tau_{\text{total}} &= (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2) \alpha \\ &= \left(\sum_{i=1}^n m_i r_i^2 \right) \alpha \end{aligned}$$

or $\tau = I\alpha$ (3.18)

where I is the moment of inertia of the body and is expressed as

$$I = \sum_{i=1}^n m_i r_i^2 \quad \dots \dots \dots (3.19)$$

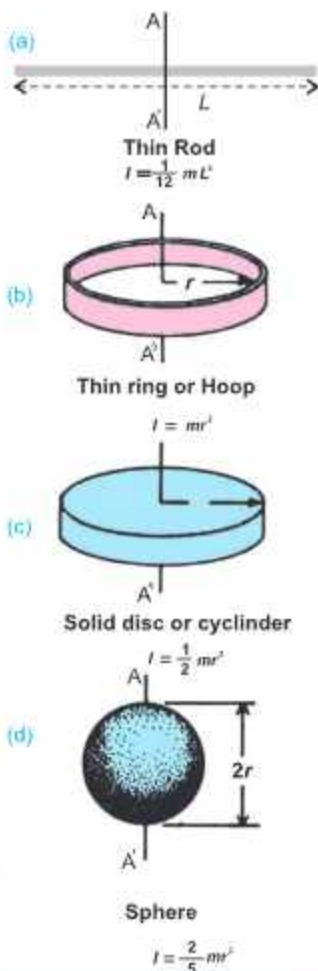
Do you know?



Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

For your information

Moments of inertia of various bodies about axis AA'.



3.5 ANGULAR MOMENTUM

We have already seen that linear momentum plays an important role in translational motion of bodies. Similarly, another quantity known as angular momentum has important role in the study of rotational motion.

A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum \mathbf{L} of a particle of mass m moving with velocity \mathbf{v} and momentum \mathbf{p} (Fig. 3.17) relative to the origin O is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \quad \dots \dots \dots (3.20)$$

where \mathbf{r} is the position vector of the particle at that instant relative to the origin O . Angular momentum is a vector quantity. Its magnitude is:

$$L = rp \sin\theta = mrv \sin\theta$$

where θ is the angle between \mathbf{r} and \mathbf{p} . The direction of \mathbf{L} is perpendicular to the plane formed by \mathbf{r} and \mathbf{p} and its sense is given by the right hand rule of vector product. SI unit of angular momentum is $\text{kg m}^2 \text{s}^{-1}$ or J s.

If the particle is moving in a circle of radius r with uniform angular velocity ω , then angle between r and tangential velocity is 90° . Hence,

$$L = mrv \sin 90^\circ = mrv$$

But $v = r\omega$ Hence $L = mr^2\omega$

Now consider a symmetric rigid body rotating about a fixed axis through the centre of mass as shown in Fig 3.18. Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω . The magnitude of the angular momentum of the particle of mass m_i is $m_i v_i r_i$ about the origin O . The direction of \mathbf{L} is the same as that of ω . Since $v_i = r_i \omega$, the angular momentum of the i th particle is $m_i r_i^2 \omega$. Summing this over all particles gives the total angular momentum of the rigid body.

$$\mathbf{L} = \left(\sum_{i=1}^n m_i r_i^2 \right) \omega = I\omega$$

where I is the moment of inertia of the rigid body about the axis of rotation.

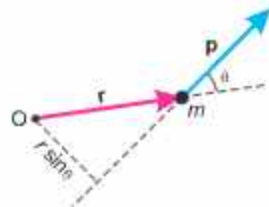
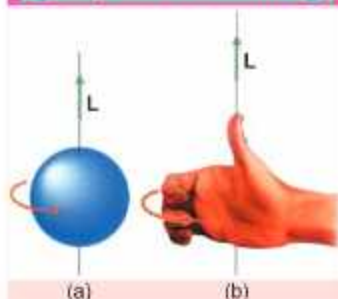


Fig. 3.17

For your information



The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).



Fig. 3.18

Example 3.5 The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7 s).

Solution To find the Earth's orbital angular momentum, we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r , it travels a

distance of $2\pi r$ in one year. Its orbital speed v_o is thus, $v_o = \frac{2\pi r}{t}$

Orbital angular momentum of the Earth = $L_o = mv_o r$

$$\begin{aligned} L &= \frac{2\pi r^2 m}{t} \\ &= \frac{2\pi (1.50 \times 10^{11} \text{ m})^2 \times (6.00 \times 10^{24} \text{ kg})}{3.16 \times 10^7 \text{ s}} \end{aligned}$$

$$L = 2.67 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$$

The sign is positive because the revolution is counter clockwise.

3.6 LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$\mathbf{L}_{\text{total}} = \mathbf{L}_1 + \mathbf{L}_2 + \dots = \text{constant}$$

The law of conservation of angular momentum is one of the fundamental principles of Physics. It has been verified from the cosmological to the sub microscopic level. The effect of the law of conservation of angular momentum is readily apparent if a single isolated spinning body alters its moment of inertia.

If a body of moment of inertia I_1 , spinning with angular speed ω_1 , alters its moment of inertia to I_2 , then its angular speed ω_2 also changes so that its angular momentum remains constant.

Hence $I_1 \omega_1 = I_2 \omega_2$

The angular momentum is a vector quantity with direction along the axis of rotation. Hence, the direction of angular momentum along the axis of rotation also remains fixed. This is illustrated by the fact given below:

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

Do you know?

If you try to sit on a bike at rest, it falls. But if the bike is moving, the angular momentum of the spinning wheel resists any tendency to change and helps to keep the bike upright and stable.

Do you know?



The ball's speed increases as the string wraps around the finger.

This fact is of great importance for the Earth as it moves around the Sun. No other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

Examples of conservation of angular momentum

A man diving from a diving board

A diver jumping from a springboard has to take a few somersaults in air before touching the water surface, as shown in Fig.3.19. After leaving the springboard, he curls his body by rolling arms and legs in. Due to this, his moment of inertia decreases, and he spins in midair with a large angular velocity. When he is about to touch the water surface, he stretches out his arms and legs. He enters the water at a gentle speed and gets a smooth dive. This is an example of the law of conservation of angular momentum.

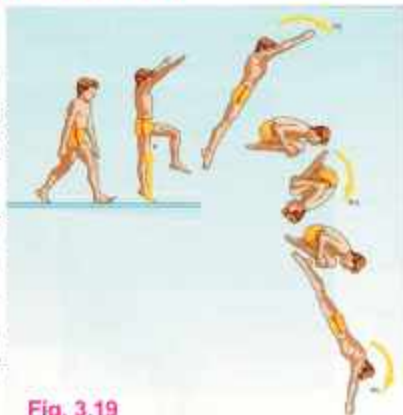


Fig. 3.19

A man diving from a diving board.

The spinning ice skater

An ice skater as shown in Fig.3.20 can increase his angular velocity by folding arms and bringing the stretched leg close to the other leg. By doing so, he decreases his moment of inertia. As a result, angular speed increases. When he stretches his hands and a leg outward, the moment of inertia increases and hence angular velocity decreases.

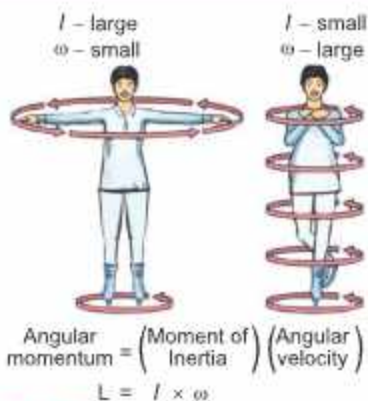


Fig. 3.20

An ice skater using angular momentum

A person holding some weight in his hands standing on a turntable.

A person is standing on a turntable with heavy mass (dumb-bell) in his hands stretched out on both sides as shown in Fig. 3.21. As he draws his hands inward, his

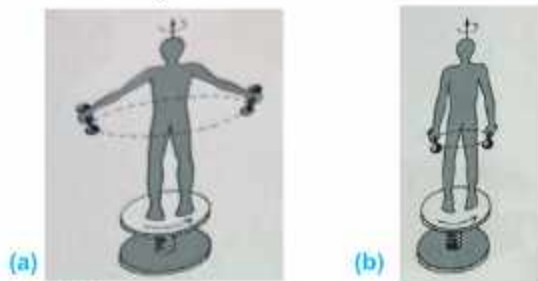


Fig. 3.21

Man with masses in his hands on a turntable. Conservation of angular momentum requires that as the man pulls his arms in, the angular velocity increases.

For your information

It has been noticed that when ice on the polar caps of Earth melts and water flows away in the form of river; the moment of inertia of water and hence that of Earth about its axis of rotation increases due to conservation of angular momentum. Hence, the angular velocity of Earth will decrease, therefore, the duration of day increases slightly.

angular speed at once increases. This is because the moment of inertia decreases on drawing the hands inwards, which increases the angular speed.

Flywheel

Flywheel is a mechanical device which consists of a heavy wheel with an axle (Fig. 3.22). It is used to store rotational energy, smooth out output fluctuations and provides stability in a wide range of applications such as bicycles and other vehicles, industrial machinery, gyroscopes, ships and spacecrafts.

Do you know?

The flywheel called the balance wheel regulates the time keeping mechanism in mechanical clocks and watches by maintaining controlled oscillations rate.



Fig. 3.22: A flywheel

When a fly wheel spins, its angular momentum resists changes to its orientations, maintaining stability. This is useful in systems that need precise control over their orientation without external interference.

The Gyroscope

A gyroscope is a device which is used to maintain its orientation relative to the Earth's axis or resists changes in its orientation. It consists of a mounted flywheel pivoted in supporting rings as shown in Fig. 3.23. It works on the basis of law of conservation of angular momentum due to its large moment of inertia. When the gyroscope spins at a large angular speed, it gains large angular momentum. It is then difficult to change the orientation of the gyroscope's rotational axis due to its large moment of inertia. A change in orientation requires a change in its angular momentum. To change the direction of a large angular momentum, a corresponding large torque is required. Even if gyroscope is tilted (Fig. 3.24), it still keeps levitated without falling. Hence, it is a reason why a gyroscope can be used to maintain orientation. The

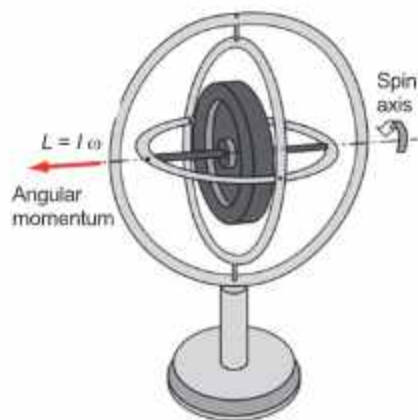


Fig. 3.23: The gyroscope



Fig. 3.24

Point to ponder!



Why does the coasting rotating system slow down as water drips into the beaker?

main applications of gyroscope are in the guiding system of aeroplanes, submarines and space vehicles in order to maintain a specific direction in space to keep steady course.

Point to ponder!

Planets move around the Sun in elliptical orbits with Sun situated at one of its foci, thus, distance of a planet from the Sun is not constant when it is nearer the Sun. Its orbital velocity increases automatically. Why?

QUESTIONS**Multiple Choice Questions**

Tick (✓) the correct answer.

- 3.1 The ratio of angular speed of minute's hand and hour's hand of a watch is:
(a) 1 : 6 (b) 6 : 1
(c) 1 : 12 (d) 12 : 1
- 3.2 A body travelling in a circle at constant speed:
(a) has constant velocity (b) has an inward radial acceleration
(c) is not accelerated (d) has an outward radial acceleration
- 3.3 A stone at the end of long string is whirled in vertical circle at a constant speed. The tension in the string will be minimum when the stone is:
(a) at the top of the circle (b) half way down
(c) at the bottom of circle (d) any where in the circle
- 3.4 Every point of rotating rigid body has:
(a) same angular velocity (b) same linear velocity
(c) same linear acceleration (d) same linear distance
- 3.5 The minimum velocity necessary to put a satellite into the orbit is called:
(a) terminal velocity (b) critical velocity
(c) artificial velocity (d) angular velocity
- 3.6 An astronaut is orbiting around the Earth in a large capsule. Then,
(a) he will be in a state of weightlessness with respect to capsule
(b) he is freely falling towards the Earth
(c) a ball projected at an angle has a straight line path as observed by him
(d) all the above
- 3.7 An object in uniform circular motion makes 10 revolutions in 2 seconds. Which of the following statement is true?
(a) Its period is 2.0 s (b) Its period is 20 s
(c) Its frequency is 5 Hz (d) Its frequency is 0.2 Hz

- 3.8 A man inside the artificial satellite feels weightlessness because the force of attraction due to the Earth is:
- zero at pole
 - balanced by the force of attraction due to the moon
 - equal to the centripetal force
 - non-effective due to some particular design of the satellite
- 3.9 A bottle of soda water is grasped from the neck and swung briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?
- Near the bottom
 - In the middle of bottle
 - Bubbles remain distributed throughout the volume of the bottle.
 - Near the neck of the bottle
- 3.10 The moment of inertia of body depends upon:
- mass of the body and its distribution about axis of rotation
 - volume of the body
 - kinetic energy of the body
 - angular momentum of the body

Short Answer Questions

- 3.1 State second law of motion in case of rotation.
- 3.2 What is the effect of changing the position of a diver while diving in the pool?
- 3.3 How do we get butter from the milk by using centrifuge?
- 3.4 Mass is a measure of inertia in linear motion. What is its analogue in rotational motion? Describe briefly.
- 3.5 Why is it harder for a car to take turn at higher speed than at lower speed?
- 3.6 What are the benefits of using rear wheels of heavy vehicles consisted of double tyres?
- 3.7 When a moving car turns around a corner to the left, in what direction do the occupants tend to fall? Explain briefly.
- 3.8 Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?
- 3.9 How does an astronaut feel weightlessness while orbiting from the Earth in a spaceship?

Constructed Response Questions

- 3.1 If angular velocity of different particles of a rigid body is constant, will the linear velocity of these particles be also constant?
- 3.2 A loaf of bread is lying on rotating plate. A crow takes away the loaf of bread and the rotation of the plate increases. Why?

- 3.3 Why do we tumble when we take the sharp turn with large speed?
- 3.4 What will be time period of a simple pendulum in an artificial satellite at a certain height?
- 3.5 Is the motion of a satellite in its orbit, uniform or accelerated?
- 3.6 What are the advantages that radian has been preferred as SI unit to degree?
- 3.7 In uniform circular motion, what are the average velocity and average acceleration for one revolution? Explain.
- 3.8 In a rainstorm with a strong wind, what determines the best position to hold an umbrella?
- 3.9 A ball is just supported by a string without breaking. If it is whirled in a vertical circle, it breaks. Explain why.
- 3.10 How is the centripetal force supplied in the following cases:
- a satellite orbiting around the Earth?
 - a car taking a turn on a level road?

Comprehensive Questions

- 3.1 What is meant by angular momentum? Explain the law of conservation of angular momentum with daily life examples.
- 3.2 Show that orbital angular momentum is; $L = I\omega$.
- 3.3 Define moment of inertia. Prove that torque acting on rotating rigid body is equal to the product of its moment of inertia and angular acceleration.
- 3.4 What are artificial satellites? Calculate the minimum time period necessary to put a satellite into the orbit near the surface of the Earth.
- 3.5 Define orbital velocity and derive an expression for the same.
- 3.6 Write a note on artificial gravity. Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.
- 3.7 Prove that; (i) $v = r\omega$ and (ii) $a = r\alpha$

Numerical Problems

- 3.1 A laser beam is directed from the Earth to the moon. The beam spreads over a diameter of 2.50 m at the moon surface. What is divergence angle of the beam? The distance of moon from the Earth is 3.8×10^8 m. (Ans: 6.6×10^{-9} rad)
- 3.2 A car is moving with a speed of 108 km h^{-1} . If its wheel has a diameter of 60 cm, find its angular speed in rad s^{-1} and rev s^{-1} . (Ans: 100 rad s^{-1} , 16 rev s^{-1})
- 3.3 An electric motor is running at $1800 \text{ rev min}^{-1}$. On switching off, it comes to rest in 20 s. If angular retardation is uniform, find the number of revolutions it makes before stopping. (Ans: 300 rev)

- 3.4 A string 0.5 m long holding a stone can withstand maximum tension of 35.6 N. Find the maximum speed at which a stone of 0.5 kg mass can be whirled with it in a vertical circle. (Ans: 5.5 m s^{-1})
- 3.5 The flywheel of an engine is rotating at $2100 \text{ rev min}^{-1}$ when the power source is shut off. What torque is required to stop it in 3 minutes? The moment of inertia of the flywheel is 36 kg m^2 . (Ans: 44 N m)
- 3.6 What is the moment of inertia of a 200 kg sphere whose diameter is 60 cm? (Ans: 7.2 kg m^2)
- 3.7 A satellite is orbiting the Earth at an altitude of 200 km. Assuming the Earth's radius is 6400 km, calculate the orbital speed of the satellite. (Ans: 7.77 km s^{-1})
- 3.8 A space station has a radius of 20 m and rotates at an angular velocity of 0.5 rad s^{-1} . What is the artificial gravity experienced by the astronauts on the space station? (Ans: 5 m s^{-2})
- 3.9 A bicycle wheel has an angular momentum of $10 \text{ kg m}^2 \text{ s}^{-1}$ and angular velocity of 2 rad s^{-1} . Find the value of its moment of inertia. (Ans: 5 kg m^2)
- 3.10 A diver comes off a board with arms straight up and legs straight down, giving him a moment of inertia of 18 kg m^2 about his rotation axis. Then tucks into a small ball, decreasing his moment of inertia to 3.6 kg m^2 . While tucked, he makes two complete rotations in 1.0 second. If he had not tucked at all, how many revolutions would he have made in 1.5 s from board to water? (Ans: 0.6 rev)

Learning Objectives

After studying this chapter, the students will be able to:

- ◆ Derive the formula for kinetic energy [using the equations of motion]
- ◆ Derive an expression for absolute potential energy of a body at a certain position in the gravitational field [including escape velocity]
- ◆ Deduce the work done from force-displacement graph.
- ◆ Differentiate between conservative and non-conservative forces.
- ◆ State and use the work - energy theorem in a resistive medium to solve problems.

Work is often thought in terms of physical or mental effort. In Physics, however, the term work involves two things (i) force, and (ii) displacement. We shall begin with a simple situation in which work is done by a constant force.

4.1 WORK DONE BY A CONSTANT FORCE

Let us consider an object which is being pulled by a constant force F . The force displaces the object through a displacement d in the direction of force. In such a case, work W is defined as the product of the magnitude of the force F and magnitude of the displacement d . This can be written as:

$$W = Fd \quad (4.1)$$

Equation (4.1) shows that if the displacement is zero, no work is done even if a large force is applied. For example, pushing on a wall may tire your muscles, but work done is zero as shown in Fig. 4.1.

The force applied on a body may not always be in the direction of force as shown in Fig. (4.2). If the force F makes an angle θ with the displacement d (Fig. 4.3), the work done is equal to the product of the component of force along the direction of the displacement and the magnitude of displacement. Then



Fig. 4.1



Fig. 4.2

$$W = (F \cos \theta) d = Fd \cos \theta \quad \dots\dots\dots (4.2)$$

$$\text{or} \quad W = \mathbf{F} \cdot \mathbf{d} \quad \dots\dots\dots (4.3)$$

Equation (4.3) shows that work is a scalar quantity. The unit of work is joule (J). From Eq. (4.1), we have

$$1 \text{ J} = 1 \text{ N m}$$

When a constant force acts through a distance d , the event can be plotted on a simple graph (Fig. 4.4). The distance is normally plotted along x-axis and the force along y-axis. As the force does not vary, in this case, the graph will be a horizontal straight line. If the constant force F (newton) and the displacement d (metre) are in the same direction, then the work done is Fd (joule). Clearly shaded area in Fig. 4.4 is also Fd . Hence, the area under a force-displacement curve can be taken as to represent the work done by the constant force. In case the force F is not in the direction of displacement, the graph is plotted between $F \cos \theta$ and d .

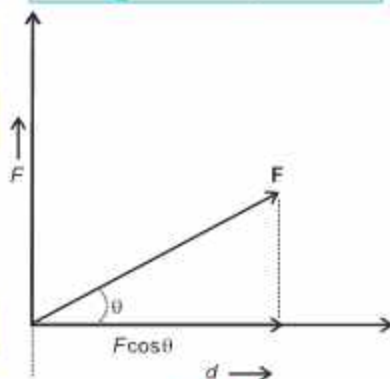


Fig. 4.3

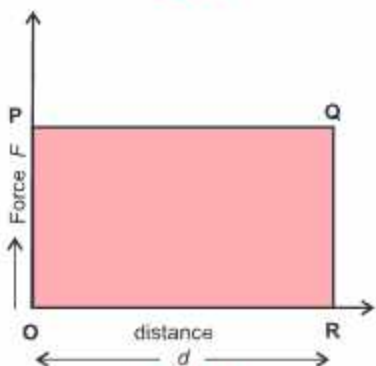


Fig. 4.4

From the definition of work, we find that:

- (i) Work is a scalar quantity.
- (ii) If $\theta < 90^\circ$, work is done and it is said to be positive work.
- (iii) If $\theta = 90^\circ$, no work is done.
- (iv) If $\theta > 90^\circ$, the work done is said to be negative.
- (v) SI unit of work is N m, also known as joule (J).

4.2 WORK DONE BY A VARIABLE FORCE

In many cases, the force does not remain constant during the process of doing work. For example, as a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre. Similarly, the force exerted by a spring increases with the amount of stretch. How can we calculate the work done in such situations?

Figure 4.5 shows the path of a particle in the xy plane as it moves from point P to point Q. The path has been divided into n short intervals of displacements $\Delta \mathbf{d}_1, \Delta \mathbf{d}_2, \dots, \Delta \mathbf{d}_n$ and $\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_n$ are the forces acting during these intervals, respectively.

During each small interval, the force is supposed to be approximately constant. So, the work done for the first interval can then be written as

$$\Delta W_1 = \mathbf{F}_1 \cdot \Delta \mathbf{d}_1 = F_1 \cos \theta_1 \Delta d_1$$

and in the second interval

$$\Delta W_2 = \mathbf{F}_2 \cdot \Delta \mathbf{d}_2 = F_2 \cos \theta_2 \Delta d_2$$

and so on. The total work done in moving the object can be calculated by adding all these terms.

$$W_{\text{total}} = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n$$

$$= F_1 \cos \theta_1 \Delta d_1 + F_2 \cos \theta_2 \Delta d_2 + \dots + F_n \cos \theta_n \Delta d_n$$

or
$$W_{\text{total}} = \sum_{i=1}^n F_i \cos \theta_i \Delta d_i \quad \dots \dots \dots (4.4)$$

We can examine this graphically by plotting $F \cos \theta$ versus d as shown in Fig. 4.6. The displacement d has been sub-divided into the same n equal intervals. The value of $F \cos \theta$ at the beginning of each interval is indicated in the figure.

Now the i^{th} shaded rectangle has an area $F_i \cos \theta_i \Delta d_i$ which is the work done during the i^{th} interval. Thus, the work done given by Eq. 4.4 equals the sum of the areas of all the rectangles. If we sub-divide the distance into a large number of intervals so that each Δd becomes very small, the work done given by Eq. 4.4 becomes more accurate. If we let each Δd to approach zero, then we obtain an exact result for the work done, such as:

$$W_{\text{total}} = \text{Limit}_{\Delta d \rightarrow 0} \sum_{i=1}^n F_i \cos \theta_i \Delta d_i \quad \dots \dots \dots (4.5)$$

If this limit Δd approaches zero, the total area of all the rectangles (Fig. 4.6) approaches the area between the $F \cos \theta$ versus d curve and x-axis from P to Q as shown shaded in Fig. 4.7.

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos \theta$ versus d curve between the two points P and Q as shown in Fig. 4.7.

Example 4.1 A force F acting on an object varies with distance d as shown in Fig. 4.8. Calculate the work done by the force as the object moves from $d = 0$ to $d = 6$ m.

Solution The work done by the force is equal to the total area under the curve from $d = 0$ to $d = 6$ m. This area is equal to the area of the rectangular section from $d = 0$ to

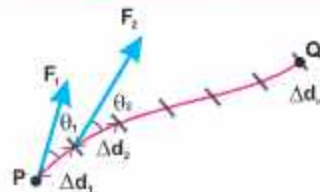


Fig. 4.5
A particle acted upon by a variable force, moves along the path shown from point P to point Q.

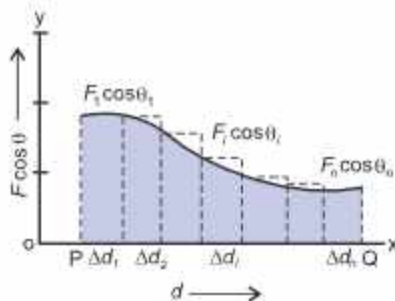


Fig. 4.6

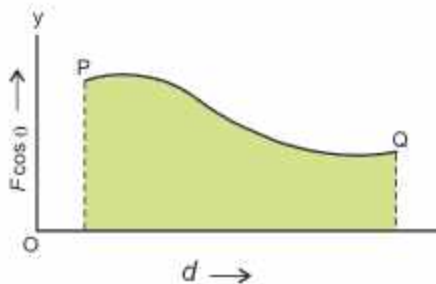


Fig. 4.7

$d = 4$ m, plus the area of triangular section from $d = 4$ m to $d = 6$ m.

Hence

Work done represented by the area of rectangle = $4 \text{ m} \times 5 \text{ N}$

$$= 20 \text{ N m} = 20 \text{ J}$$

Work done represented by the area of triangle = $\frac{1}{2} \times 2 \text{ m} \times 5 \text{ N}$

$$= 5 \text{ N m} = 5 \text{ J}$$

Therefore, the total work done

$$= 20 \text{ J} + 5 \text{ J} = 25 \text{ J}$$

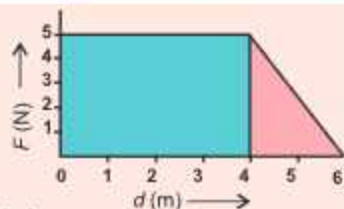


Fig. 4.8

4.3 CONSERVATIVE AND NON-CONSERVATIVE FORCES

Conservative Forces

The space around the Earth in which its gravitational force acts on a body is called the gravitational field. When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive. If the displacement is against the gravitational force, the work is said to be negative.

There is an interesting property of the gravitational force that when an object is moved from one place to another, the work done by the gravitational force does not depend on the choice of the path. Let us explore it.

Consider an object of mass m being displaced with constant speed from point A to B along various paths in the presence of a gravitational force (Fig. 4.9). In this case, the gravitational force is equal to the weight mg of the object.

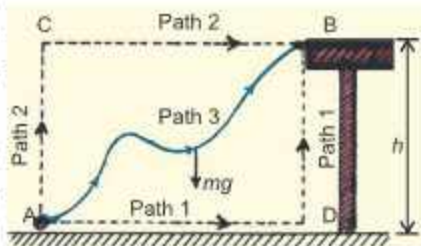


Fig. 4.9

The work done by the gravitational force along the path 1 (ADB) can be split into two parts (path AD and path DB). The work done along AD is zero, because the weight mg is perpendicular to this path, whereas the work done along DB is $(-mgh)$ because the direction of mg is opposite to that of the displacement i.e., $\theta = 180^\circ$. Hence, the work done in displacing a body from A to B through path 1 is:

$$W_{ADB} = 0 + (-mgh) = -mgh$$

If we consider the path 2 (ACB), the work done along AC is also $(-mgh)$. Since the work done along CB is zero, therefore,

$$W_{ACB} = -mgh + 0 = -mgh$$

Now consider path 3, i.e., a curved one. Imagine the curved path to be broken down into a

series of horizontal and vertical steps as shown in Fig. 4.10. There is no work done along the horizontal steps, because mg is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements. During the segment CD, mg is not negative; it is positive. But here all Δy elements are negative, so the products of mg and Δy for all the elements will again be negative. Therefore, we can write:

$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \Delta y_3 + \dots + \Delta y_n)$$

As $\Delta y_1 + \Delta y_2 + \Delta y_3 + \dots + \Delta y_n = h$

Hence $W_{AB} = -mgh$

The net amount of work done along the curved path AB is still $(-mgh)$. We conclude from the above discussion that:

Work done by gravitational force is independent of the path followed.

Can you prove that the work done, along a closed path, such as ACBA or ADDBA (Fig. 4.9), by the gravitational force is zero?

If the work done by a force in moving an object between two points is independent of the path followed or the work done in a closed path be zero, the force is called a conservative force.

As shown above, the gravitational force is a conservative force, other examples of conservative force are electrostatic force and elastic spring force.

Non-Conservative Forces

All types of forces are not conservative forces.

A force is non-conservative if the work done by it in moving an object between two points or in a closed path depends on the path of motion.

The kinetic frictional force is a non-conservative force. When an object slides over a surface, the kinetic frictional force always acts opposite to the motion and does negative work equal in magnitude to the frictional force multiplied by the length of the path. Thus, greater amount of work is done over a longer path between any two points. Hence, the work depends on the choice of path. Moreover, the total work done by a non-conservative force in a closed path is not zero.

Other examples of non-conservative force are air resistance, tension in a string, normal force and propulsion force of a rocket.

4.4 POWER

In the definition of work, it is not clear, whether the same amount of work is done in one second or in one hour. The rate, at which work is done, is often of interest in practical applications.

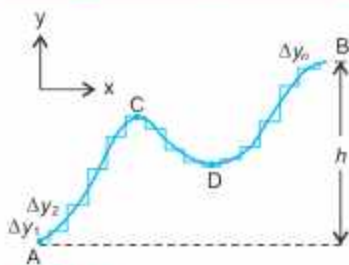


Fig. 4.10

A smooth path may be replaced by a series of infinitesimal x and y displacements. Work is done only during the y displacements.

Power is the measure of the rate at which work is being done.

If work ΔW is done in a time interval Δt , then the average power P_{av} during the interval Δt is defined as:

$$P_{av} = \frac{\Delta W}{\Delta t} \dots\dots\dots (4.6)$$

If work is expressed as a function of time, the instantaneous power P at any instant is defined as:

$$P = \text{Limit}_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} \dots\dots\dots (4.7)$$

where ΔW is the work done in short interval of time Δt .

Since $\Delta W = \mathbf{F} \cdot \Delta \mathbf{d}$

Hence $P = \frac{\mathbf{F} \cdot \Delta \mathbf{d}}{\Delta t} = \mathbf{F} \cdot \frac{\Delta \mathbf{d}}{\Delta t}$

Since $\frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v}$

Hence $P = \mathbf{F} \cdot \mathbf{v} \dots\dots\dots (4.8)$

Thus, power may also be defined as the scalar product of \mathbf{F} and \mathbf{v} .

The SI unit of power is watt, defined as one joule of work done in one second.

Sometimes, for example, in the electrical measurements, the unit of work is expressed as watt second. However, a commercial unit of electrical energy is kilowatt-hour.

One kilowatt-hour is the work done in one hour by an agency whose power is one kilowatt.

Therefore $1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ s}$

or $1 \text{ kWh} = 3.6 \times 10^6 \text{ J} = 3.6 \text{ MJ}$

For your information

Approximate Powers	
Device	Power (W)
Jumbo Jet Aircraft	1.3×10^8
Car at 90 km h ⁻¹	1.1×10^5
Electric heater	2×10^3
Coloured TV	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

Example 4.2 A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Solution Work done = mgh

$$\text{Power} = \frac{mgh}{t}$$

$$P = \frac{70 \text{ kg} \times 9.8 \text{ m s}^{-2} \times 4.5 \text{ m}}{4 \text{ s}}$$

$$P = 7.7 \times 10^2 \text{ kg m}^2 \text{ s}^{-3} = 7.7 \times 10^2 \text{ W}$$

Do you know?

It takes about $9 \times 10^8 \text{ J}$ of energy to make a car and the car then uses about $1 \times 10^{12} \text{ J}$ of energy from petrol in its life time.

4.5 ENERGY

Energy of a body is its capacity to do work. There are two basic forms of energy:

- (i) Kinetic energy (ii) Potential energy

Kinetic energy is the energy possessed by a body due to its motion and potential energy is the energy possessed by a body due to its changed position.

The kinetic energy and the potential energy both are the kinds of mechanical energy.

Kinetic Energy

Let us derive a formula for the kinetic energy of a moving body. Consider a car running with a constant speed on a road. If its engine is switched OFF, it will still cover some distance before stopping. As long as it is moving, it is doing work against the force of friction of the road. In other words, during this interval, it will exert a force equal in magnitude to the force of friction f . Let the distance travelled before coming to rest be d , then the work done by the car would be fd . This work is done by the car due to its motion. The ability of a body to do work due to its motion is its kinetic energy. Therefore, kinetic energy of the car is equal to fd . The acceleration can be found by using Newton's second law of motion, i.e.,

$$F = ma$$

As the car slows down and finally stops, its acceleration a is negative because it is produced by force of friction f acting opposite to the direction of motion. Thus,

$$f = -ma$$

or
$$a = -\frac{f}{m}$$

We can now determine the value of (fd) by using the third equation of motion, i.e.:

$$2aS = v_f^2 - v_i^2$$

Here, Initial velocity $v_i = v$

Final velocity $v_f = 0$

Distance $S = d$

Acceleration $a = -\frac{f}{m}$

Putting values in the above equation of motion, we have

$$2 \times \left(-\frac{f}{m}\right) d = 0 - v^2$$

$$fd = \frac{1}{2}mv^2$$

As fd is equal to the kinetic energy of body, therefore,

$$\text{Kinetic energy} = \frac{1}{2}mv^2 \quad (4.9)$$

Since, kinetic energy is equal to work which the body is capable of doing, so the unit of kinetic energy must be that of work, i.e. joule (J).

Example 4.3 A car weighing 18620 N is running with a speed of 16 m s^{-1} . Brakes are applied and it is brought to rest in a distance of 80 m. Determine the average force of

For your information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^9
Burning 1 litre petrol	5×10^7
K.E. of a car at 90 km h ⁻¹	1×10^6
Running Person at 10 km h ⁻¹	3×10^2
Fission of one atom of uranium	1.8×10^{11}
K.E. of a molecule of air	6×10^{-21}

friction acting on it.

Solution

Given that $v = 16 \text{ m s}^{-1}$, $d = 80 \text{ m}$, $w = 18620 \text{ N}$ and $f = ?$

The kinetic energy of the car is equal to the work done by it before stopping, i.e.,

$$\frac{1}{2}mv^2 = fd$$

Here $m = \frac{w}{g} = \frac{18620 \text{ N}}{9.8 \text{ m s}^{-2}} = 1900 \text{ kg}$

Putting the value in the above equation, we have

$$\frac{1}{2} \times 1900 \text{ kg} \times (16 \text{ m s}^{-1})^2 = f \times 80$$

or $f = 3040 \text{ N}$

Potential Energy

The potential energy is possessed by a body because of its position in a force field, e.g. gravitational field or because of its constrained state.

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Absolute Potential Energy

The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero.

The relation for the calculation of the work done by the gravitational force or potential energy is mgh , which is true only near the surface of the Earth where the gravitational force is nearly constant. But if the body is displaced through a large distance in space, let it be from point 1 to N (Fig. 4.11) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance.

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If r_1 and r_2 are the distances of points 1 and 2 respectively, from the centre O of the Earth (Fig. 4.11.), the

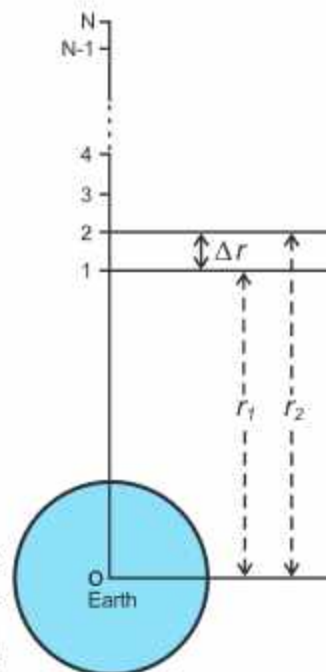


Fig. 4.11

work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as below. The distance between the centre of this step and centre of the Earth will be:

$$r = \frac{r_1 + r_2}{2}$$

As $r_2 - r_1 = \Delta r$ then $r_2 = r_1 + \Delta r$

$$\text{Hence } r = \frac{r_1 + r_1 + \Delta r}{2} = r_1 + \frac{\Delta r}{2} \dots\dots\dots (4.10)$$

The gravitational force F at the centre of this step is:

$$F = G \frac{Mm}{r^2} \dots\dots\dots (4.11)$$

where m = mass of an object , M = mass of the Earth
and G = Gravitational constant

Squaring Eq. 4.10

$$r^2 = \left(r_1 + \frac{\Delta r}{2} \right)^2$$

$$r^2 = r_1^2 + 2r_1 \frac{\Delta r}{2} + \left(\frac{\Delta r}{2} \right)^2$$

As $(\Delta r)^2 \ll r_1^2$, so $(\Delta r)^2$ can be neglected as compared to r_1^2 .

$$\text{Hence } r^2 = r_1^2 + r_1 \Delta r$$

Putting the value of $\Delta r = r_2 - r_1$

$$r^2 = r_1^2 + r_1 (r_2 - r_1) = r_1 r_2$$

Hence, Eq. 4.11 becomes

$$F = G \frac{Mm}{r_1 r_2} \dots\dots\dots (4.12)$$

As this force is assumed to be constant during the interval Δr , so the work done is:

$$W_{1 \rightarrow 2} = \mathbf{F} \cdot \Delta \mathbf{r} = F \Delta r \cos 180^\circ = -GMm \frac{\Delta r}{r_1 r_2}$$

The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force. Putting the value of Δr , we have

$$W_{1 \rightarrow 2} = -GMm \frac{r_2 - r_1}{r_1 r_2}$$

$$\text{or } W_{1 \rightarrow 2} = -GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Do you know?



There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

Similarly, the work done during the second step in which the body is displaced from point 2 to 3 is:

$$W_{2 \rightarrow 3} = -GMm \left(\frac{1}{r_2} - \frac{1}{r_3} \right)$$

and the work done in the last step is:

$$W_{N-1 \rightarrow N} = -GMm \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right)$$

Tidbit

More coal has been used since 1945 than was used in the whole of history before that.

Hence, the total work done in displacing a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + \dots + W_{N-1 \rightarrow N}$$

$$= -GMm \left[\left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \left(\frac{1}{r_2} - \frac{1}{r_3} \right) + \dots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right) \right]$$

On simplification, we have

$$W_{total} = -GMm \left(\frac{1}{r_1} - \frac{1}{r_N} \right)$$

If the point N is situated at an infinite distance from the Earth, then

$$r_N = \infty, \quad \text{so} \quad \frac{1}{r_N} = \frac{1}{\infty} = 0$$

$$\text{Hence} \quad W_{total} = -\frac{GMm}{r_1}$$

This total work by definition is the absolute potential energy (P.E) as stated earlier represented by U .

$$U = -\frac{GMm}{r}$$

This is also known as the absolute value of gravitational potential energy of a body at a distance r from the centre of the Earth.

Note that when r increases, U becomes less negative i.e., U increases. It means when we raise a body above the surface of the Earth, its P.E. increases. Therefore, if we want to raise the body up to infinite distance, we will have to do work on it equal to $\frac{GMm}{R}$, so that its P.E. becomes zero.

Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth), so

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R} \quad \dots \quad (4.13)$$

The negative sign shows that the Earth's gravitational field for mass m is attractive. The above expression gives the work or the energy required to take the body out of the

Earth's gravitational field, where its potential energy with respect to Earth is zero.

4.6 ESCAPE VELOCITY

It is our daily life experience that an object projected upward comes back to the ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape from the influence of gravity.

The initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity.

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of the Earth.

$$\text{Initial K.E.} = \frac{1}{2} m v_{\text{esc}}^2$$

We know that the work done in lifting a body from Earth's surface to an infinite distance is equal to the increase in potential energy.

$$\text{Increase in P.E.} = 0 - \left(-G \frac{Mm}{R}\right) = G \frac{Mm}{R}$$

where M and R are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial K.E. of the body is equal to increase in P.E. Then

$$\frac{1}{2} m v_{\text{esc}}^2 = G \frac{Mm}{R}$$

$$\text{or} \quad v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \quad \dots\dots\dots (4.14)$$

$$\text{As} \quad g = \frac{GM}{R^2} \quad \text{or} \quad gR = \frac{GM}{R}$$

$$\text{Hence} \quad v_{\text{esc}} = \sqrt{2gR} \quad \dots\dots\dots (4.15)$$

The value of v_{esc} comes out to be approximately 11 km s^{-1} .

4.7 WORK-ENERGY THEOREM

Whenever work is done on a body, it increases its energy. For example, if a force F acts on a body of mass m , initially moving with velocity v_i , through a distance d and increases its velocity to v_f , then the acceleration produced will be:

$$2ad = v_f^2 - v_i^2$$

For your information

Some Escape speeds (km s^{-1})

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

$$\text{or } a = \frac{1}{2d}(v_f^2 - v_i^2) \quad \dots\dots\dots (4.16)$$

From the second law of motion:

$$F = ma$$

$$\text{or } a = \frac{F}{m} \quad \dots\dots\dots (4.17)$$

Comparing Eqs. 4.16 and 4.17, we have

$$\frac{F}{m} = \frac{1}{2d}(v_f^2 - v_i^2)$$

$$\text{or } Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \dots\dots\dots (4.18)$$

This expression is the work-energy theorem. It states that:

The change in kinetic energy of an object is equal to the work done on it by a net force.

$$W = \text{Change in kinetic energy} = (K.E.)_f - (K.E.)_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This is also known as work-energy principle.

The work-energy theorem is applicable for any direction of the force relative to the displacement. For instance, an object with kinetic energy can perform work if it is allowed to push or pull on another object. In this case, the work will be taken as negative and the kinetic energy of the object will decrease. The theorem remains valid even if the force may vary from point to point.

Example 4.4 A motorcycle rider weighing 60 kg is coasting down a 24° slope. The weight of motorcycle is 30 kg. At the top of the slope, the speed of motorcycle is 3.2 m s^{-1} . If the kinetic frictional force is 100 N, what will be the speed of the motorcycle 72 m downhill?



Solution The normal force F_N is balanced by the component of weight ($mg\cos 24^\circ$) perpendicular to the slope. Let the kinetic frictional force is f , then the net force F is:

$$F = mg \sin 24^\circ - f \quad \text{where } m = \text{total mass} = 60 \text{ kg} + 30 \text{ kg} = 90 \text{ kg}$$

$$\text{or } F = (90 \text{ kg} \times 9.8 \text{ m s}^{-2} \times 0.4) - 100 \text{ N}$$

$$F = 252.8 \text{ N}$$

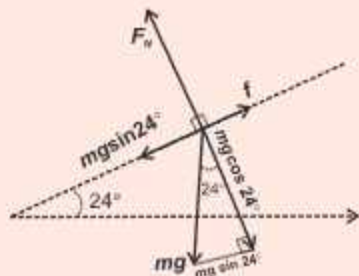
$$\text{Work done } W = Fd = 252.8 \text{ N} \times 72 \text{ m} = 18201.6 \text{ J}$$

As work is positive, so applying work – energy theorem,

$$W = (K.E.)_f - (K.E.)_i$$

From here,

$$(K.E.)_f = W + (K.E.)_i$$



Tidbits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Putting the values, we have

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_i^2$$

$$\frac{1}{2} \times 90 \text{ kg} \times v_f^2 = 18201 \text{ J} + \frac{1}{2} \times 90 \text{ kg} \times (3.2 \text{ m s}^{-1})^2$$

This gives, $v_f^2 = 414.7 \text{ m}^2 \text{ s}^{-2}$

or $v_f = \sqrt{414.7 \text{ m}^2 \text{ s}^{-2}} = 20.4 \text{ m s}^{-1}$

4.8 INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

Consider a body of mass m at rest, at a height h above the surface of the Earth as shown in Fig. 4.12. At position A, the body has $P.E. = mgh$ and $K.E. = 0$. We release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

Let us calculate $P.E.$ and $K.E.$ at the position B when the body has fallen a distance x , ignoring air friction.

Now, height from the ground is $(h-x)$, so that

$$P.E. = mg(h-x)$$

and $K.E. = \frac{1}{2}mv_B^2$

Velocity v_B , at position B, can be calculated from the relation,

$$v_f^2 = v_i^2 + 2gS$$

as $v_f = v_B$, $v_i = 0$, $S = x$

$$v_B^2 = 0 + 2gx$$

$$v_B^2 = 2gx$$

Therefore $K.E. = \frac{1}{2}m(2gx)$

$$= mgx$$

Total energy at position B = $P.E. + K.E.$

$$\text{Total energy} = mg(h-x) + mgx = mgh \quad \dots\dots (4.19)$$

At position C, just before the body strikes the Earth, $P.E. = 0$ and $K.E. = \frac{1}{2}mv_C^2$, where v_C can be found out by the following expression.

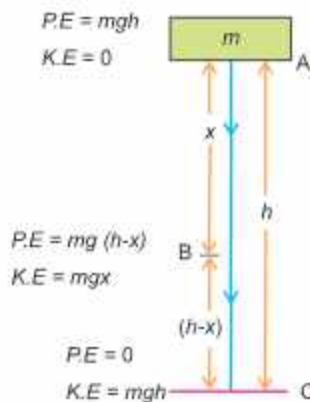


Fig. 4.12

$$v_c^2 = v_i^2 + 2gh = 2gh \quad \text{as } v_i = 0$$

$$\text{i.e.,} \quad K.E = \frac{1}{2}mv_c^2 = \frac{1}{2}m \times 2gh = mgh$$

Thus, at point C, kinetic energy is equal to the original value of the potential energy of the body. Actually, when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus, we see (Fig. 4.13) that:

$$\text{Loss in P.E.} = \text{Gain in K.E.}$$

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2) \quad \dots\dots\dots (4.20)$$

where v_1 and v_2 are the velocities of the body at the heights h_1 and h_2 respectively. This result is true only when frictional force is not considered.

If we assume that a frictional force f is present during the downward motion, then a part of P.E. is used in doing work against friction equal to fh . The remaining P.E. = $mgh - fh$ is converted into K.E.

$$\text{Hence} \quad mgh - fh = \frac{1}{2}mv^2$$

$$\text{or} \quad mgh = \frac{1}{2}mv^2 + fh \quad \dots\dots\dots (4.21)$$

$$\text{Thus} \quad \text{Loss in P.E.} = \text{Gain in K.E.} + \text{Work done against friction}$$

Conversely,

$$\text{Loss of K.E.} = \text{Gain in P.E.} + \text{Work done against friction}$$

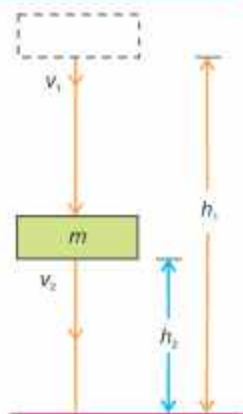


Fig. 4.13

Example 4.5

A car weighing 1100 kg is moving with a velocity of 12 m s^{-1} . When it is at point P, its engine stops. If the frictional force is 120 N, what will be its velocity at point Q? How far beyond Q will it go before coming to rest?

Solution

The kinetic energy possessed by the car at point P will partly be converted into P.E. and partly used up in doing work against friction as it reaches point Q. Therefore,

$$\text{Loss of K.E.} = \text{Gain in P.E.} + \text{Work against friction}$$



$$\frac{1}{2}m(v_i^2 - v_f^2) = wh + fd$$

$$\frac{1}{2} \times 1100 \text{ kg} (144 \text{ m}^2 \text{ s}^{-2} - v_f^2) = (1100 \text{ kg} \times 9.8 \text{ m s}^{-2} \times 1.5 \text{ m}) + 120 \text{ N} \times 24 \text{ m}$$

$$550 \text{ kg} (144 \text{ m}^2 \text{ s}^{-2} - v_f^2) = 16170 \text{ kg m}^2 \text{ s}^{-2} + 2880 \text{ kg m}^2 \text{ s}^{-2}$$

$$(144 \text{ m}^2 \text{ s}^{-2} - v_f^2) = \frac{16170 \text{ kg m}^2 \text{ s}^{-2} + 2880 \text{ kg m}^2 \text{ s}^{-2}}{550 \text{ kg}} = 34.6 \text{ m}^2 \text{ s}^{-2}$$

$$v_f^2 = 144 \text{ m}^2 \text{ s}^{-2} - 34.6 \text{ m}^2 \text{ s}^{-2} = 109.4 \text{ m}^2 \text{ s}^{-2}$$

$$\text{Velocity at point Q, } v_f = \sqrt{109.4 \text{ m}^2 \text{ s}^{-2}} = 10.5 \text{ m s}^{-1}$$

Now if the car stops at point R, then using the formula:

$$\frac{1}{2}mv^2 = fS$$

$$\frac{1}{2} \times 1100 \text{ kg} \times 109.4 \text{ m}^2 \text{ s}^{-2} = 120 \text{ kg m s}^{-2} \times S$$

$$S = 501 \text{ m approximately}$$

Example 4.6 An object of mass 3 kg falls from a height of 15 m. If it strikes the ground with a velocity of 16 m s⁻¹, calculate the average frictional force of the air.

Solution Loss of P.E = Gain in K.E. + Work done against friction

$$\therefore v_i = 0, \quad mgh = \frac{1}{2}mv^2 + fh$$

$$3 \text{ kg} \times 9.8 \text{ m s}^{-2} \times 15 \text{ m} = \frac{1}{2} \times 3 \text{ kg} (16 \text{ m s}^{-1})^2 + f \times 15 \text{ m}$$

$$441 \text{ kg m}^2 \text{ s}^{-2} = 384 \text{ kg m}^2 \text{ s}^{-2} + 15 \text{ m} \times f$$

$$\text{or } f = \frac{441 \text{ kg m}^2 \text{ s}^{-2} - 384 \text{ kg m}^2 \text{ s}^{-2}}{15 \text{ m}} = 3.8 \text{ kg m s}^{-2} = 3.8 \text{ N}$$

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

- 4.1 A 1 kg mass has potential energy of 1 joule relative to the ground when it is at a height of:
 (a) 0.102 m (b) 1 m (c) 9.8 m (d) 32 m
- 4.2 An iron sphere whose mass is 30 kg has the same diameter as an aluminium sphere whose mass is 10.5 kg. The spheres are simultaneously dropped from a cliff. When they are 10 m from the ground, they have identical:
 (a) accelerations (b) momentums (c) potential energies (d) kinetic energies

- 4.3 A body at rest may have:
(a) speed (b) velocity (c) momentum (d) energy
- 4.4 The height above the ground of a child on a swing varies from 0.5 m of his lowest point to 1.5 m at his highest point. The maximum speed of the child is approximately:
(a) 1.5 m s^{-1} (b) 4.4 m s^{-1}
(c) 9.8 m s^{-1} (d) Depends upon child's mass
- 4.5 When a ball is thrown vertically upward and then falls back to the ground, which force can be considered conservative in this scenario?
(a) Air resistance (b) Gravity
(c) Friction between ball and air (d) Contact force with hand
- 4.6 According to work-energy principle in linear motion, the work done on body is equal to:
(a) change in *K.E.* (b) change in *P.E.*
(c) zero (d) sum of *K.E.* and *P.E.*
- 4.7 Power of a lamp is 6 W. How much energy does a lamp give out in 2 min?
(a) 12 J (b) 20 J (c) 3 J (d) 720 J
- 4.8 A dry battery can deliver 3000 J of energy to a 2 W small electric motor before the battery is exhausted. For how many minutes does the battery run?
(a) 1500 min (b) 100 min (c) 50 min (d) 25 min
- 4.9 The kinetic energy acquired by a mass *m* after travelling a fixed distance from rest under the action of a constant force is directly proportional to:
(a) \sqrt{m} (b) $1/\sqrt{m}$ (c) *m* (d) independent of *m*
- 4.10 A body moves a distance of 10 m along a straight line under the action of 5 N force. If the work done is 25 J, the angle which the force makes with the direction of motion of the body is:
(a) 0° (b) 30° (c) 60° (d) 90°

Short Answer Questions

- 4.1 Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?
- 4.2 A body is being raised to a height *H* from surface of the Earth. What is the sign of work done by both (body and the Earth)? Justify.
- 4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during fall? Justify.
- 4.4 Calculate power of a crane in kilowatt which lifts a mass of 1000 kg to a height of 100 m in 20 second.

- 4.5 A trolley of mass 1500 kg carrying sand bags of 500 kg is moving uniformly with a speed of 40 km h^{-1} on a frictionless track. After some time, sand starts leaking out of whole sand bags on the road at a rate of 0.05 kg s^{-1} . What is the speed of the trolley after entire sand bags are empty?
- 4.6 Give absolute and gravitational units of work in M.K.S and C.G.S systems.
- 4.7 A body dropped from a height of H reaches the ground with a speed of $1.2 \sqrt{gH}$. Calculate work done by air friction.
- 4.8 A bicycle has a $K.E.$ of 150 J. What $K.E.$ would the bicycle have if it had:
(i) same mass but has speed double?
(ii) three times mass and was moving with one half of the speed?
- 4.9 What will be the effect on $K.E.$ of the body having mass m , moving with velocity v when its momentum becomes double? Justify.
- 4.10 Does the international space station have gravitational $P.E.$ or kinetic energy or both? Explain.

Constructed Response Questions

- 4.1 When will you say that a force is conservative? Give two conditions.
- 4.2 A light and a heavy body have same linear momentum, which one has greater $K.E.$?
- 4.3 A motorcycle is running with constant speed on a horizontal track. Is any work being done on the motorcycle, if no net force is acting on it?
- 4.4 A force acts on a ball moving with 14 m s^{-1} speed and brings its speed to 6 m s^{-1} . Has the force done positive or negative work? Explain your answer.
- 4.5 A slow moving truck can have more kinetic energy than a fast moving car. How is this possible?
- 4.6 Why work done against friction is non-conservative in nature? Explain briefly.
- 4.7 Does wind contain kinetic energy? Explain.

Comprehensive Questions

- 4.1 Define $K.E.$ and derive an expression for the same.
- 4.2 How is work done by a:
(i) constant force (ii) variable force?
- 4.3 Define conservative field. Show that gravitational field is conservative in nature.
- 4.4 What is meant by absolute $P.E.$? Derive an expression for absolute $P.E.$
- 4.5 State and explain work-energy theorem in a resistive medium.
- 4.6 Define escape velocity. Show that an expression for escape velocity can be expressed as $\sqrt{2Rg}$, where R and g denote radius of the Earth and acceleration due to gravity, respectively. Also find its numerical value near the surface of the Earth.

Numerical Problems

- 4.1 A machine gun fires 6 bullets per minute with a velocity of 700 m s^{-1} . If each bullet has a mass of 40 g, then find power developed by the gun? (Ans: 980 W)
- 4.2 A family uses 10 kW of power. Direct solar energy is incident on horizontal surface at an average rate of 300 W per square metre. If 75% of this energy can be converted into useful electrical energy, how large area is needed to supply 10 kW? (Ans: 44.44 m^2)
- 4.3 The mass of the Earth is $6.0 \times 10^{24} \text{ kg}$ and mass of the Sun is $1.99 \times 10^{30} \text{ kg}$. The Sun is 160 million km away from the Earth. Find the value of gravitational P.E. of the Earth. (Ans: $\approx -4.97 \times 10^{33} \text{ J}$)
- 4.4 An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1} . What is the frictional force acting on it? (Ans: 26 N)
- 4.5 A 75 watt fan is used for 8 hours daily for 30 days. Find:
(i) energy consumed in electrical units
(ii) electricity bill if one unit costs Rs. 22.5? [Ans: (i) 18 units (ii) Rs. 405]
- 4.6 If an object of mass 2 kg thrown up from ground reaches a height of 5 m and falls back to the Earth (neglecting air resistance), calculate:
(i) work done by gravity when the object reaches at 5 m height.
(ii) work done by gravity when the object comes back to the Earth.
(iii) total work done by gravity in upward and downward motion. Also mention physical significance of the result.
[Ans: (i) -98 J (ii) +98 J (iii) 0 J work done in a closed path in a conservative field is zero]
- 4.7 An electrical motor of one horse power is used to run a water pump. Water pump takes 15 minutes to fill a tank of 400 litres at a height of 10 m (1 hp 746 watts). Find:
(a) actual input work done by electric motor to fill the tank
(b) actual output work done [Ans: (a) 671.4 kJ, (b) 39.2 kJ]
- 4.8 A passenger just arrives at the airport and dragging his suitcase to luggage checks in at the desk. He pulls strap with a force of 200 N at an angle of 45° to the floor to displace it 50 m to the desk. Determine the value of work done by him on the suitcase. (Ans: 7 kJ)
- 4.9 A 1200 kg car is running at a speed of 40 km h^{-1} . How much power will be expended by it to accelerate at 2 m s^{-2} ? (Ans: 26.67 kW)
- 4.10 A 200 g apple is lifted to 10 m and then dropped. What is its velocity when it hits the ground? Assume that 75% of work done in lifting the apple is transferred to K.E. by the time it hits the ground. (Ans: 12.1 m s^{-1})

Learning Objectives

After studying this chapter, the students will be able to:

- ◆ Distinguish between the structures of crystalline, amorphous, and polymeric solids.
- ◆ Describe that deformation of solids in one dimension [That it is caused by a force and that in one dimension, the deformation can be tensile or compressive.]
- ◆ Define and use the terms stress, strain and the Young's modulus
- ◆ Describe an experiment to determine the Young's modulus of a metal wire.
- ◆ Describe and use the terms elastic deformation, plastic deformation and elastic limit
- ◆ Justify why and apply the fact that the area under the force-extension graph represents the work done
- ◆ Determine the elastic potential energy of a material [That is deformed within its limit of proportionality from the area under the force-extension graph. Also state and use $E_p = \frac{1}{2} kx^2$ for a material deformed within its limit of proportionality]
- ◆ State and use Archimedes' principle and flotation
- ◆ Justify how ships are engineered to float in the sea
- ◆ Define and apply the terms: steady (streamline or laminar) flow, incompressible flow and non-viscous flow as applied to the motion of an ideal fluid.
- ◆ State and use equation of continuity to solve problems
- ◆ Explain that squeezing the end of a rubber pipe results in increase in flow velocity
- ◆ Justify that the equation of continuity is a form of the principle of conservation of mass.
- ◆ Justify that the pressure difference can arise from different rates of flow of a fluid [Bernoulli effect]
- ◆ Explain and apply Bernoulli's equation for horizontal and vertical fluid flow.
- ◆ Explain why real fluids are viscous fluids.
- ◆ Describe how viscous forces in a fluid cause a retarding force on an object moving through it.
- ◆ Describe super fluidity [As the state in which a liquid will experience zero viscosity. Students should know the implications of this state e.g. this allows for super fluids to creep over the walls of containers to 'empty' themselves. It also implies that if you stir a superfluid, the vortices will keep spinning indefinitely.]
- ◆ Analyze the real-world applications of the Bernoulli effect [For example, atomizers in perfume bottles, the swinging trajectory of a spinning cricket ball and the lift of a spinning golf ball (the Magnus effect), the use of Venture ducts in filter pumps and car engineers to adjust the flow of fluid, etc.]

Materials have specific uses depending upon their characteristics and properties, such as hardness, ductility, malleability, etc. What makes a metal hard and other soft? It depends upon the structure, the particular order and bonding of atoms and molecules in a material. Similarly, the study of fluids in motion is relatively complicated but analysis can be simplified by making a few assumptions. The analysis is further simplified by the use of two important conservation principles, the conservation of mass and conservation of energy. The law of conservation of mass gives us the equation of continuity while the law of conservation of energy is the basis of Bernoulli's equation.

5.1 CLASSIFICATION OF SOLIDS

For your information

Crystalline Solids

In crystalline solids, there is a regular arrangement of atoms and molecules. The neighbours of every molecule are arranged in a regular pattern that is consistent throughout the crystal. There is, thus, an ordered structure in crystalline solids.

Most solids, like metals and ceramics have a crystalline structure. This means their atoms, molecules or ions are arranged in a regular pattern. The arrangement of molecules, atoms or ions within all types of crystalline solids can be studied using various techniques such as X-ray Diffraction (XRD) and Transmission Electron Microscopy (TEM). It should be noted that atoms, molecules or ions in a crystalline solid are not static. For example, each atom in a crystal vibrates about a fixed point with an amplitude that increases with rise in temperature. It is the average atomic positions which are perfectly ordered over large distances.

The cohesive forces between atoms, molecules or ions in crystalline solids maintain the strict long-range order in spite of atomic vibrations. For every crystal, however, there is a temperature at which the vibrations become greater than the structure suddenly breaks up, and the solid melts. The transition from solid (order) to liquid (disorder) is, therefore, abrupt or discontinuous. Every crystalline solid has a definite melting point e.g., Quartz, Calcite, Sugar, Mica, diamond, etc.

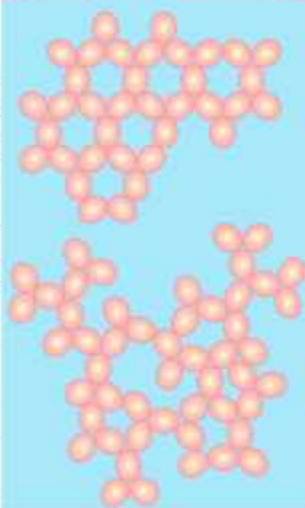
Amorphous or Glassy Solids

The word amorphous means without form or structure. Thus, in amorphous solids there is no regular arrangement of molecules like that in crystalline solids. We can, therefore, say that amorphous solids are more like liquids with the disordered structure frozen in.

For example, ordinary glass, which is a solid at ordinary temperature, has no regular arrangement of molecules. On heating, it gradually softens into a paste like state before it becomes a very viscous liquid at almost 800°C . Thus, amorphous solids are also called glassy solids. This type of solids has no definite melting point e.g., plastic, glass, fused silicon, etc.

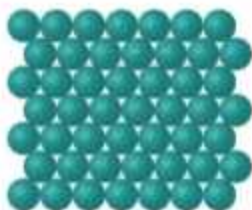
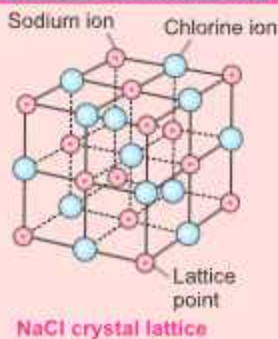
Polymeric Solids

Polymers are solid materials with a structure that is intermediate between order and disorder. They can be classified as partially or poorly crystalline solids.



Glassy and crystalline-solid—short and long-range order.

For your information



Crystalline solids

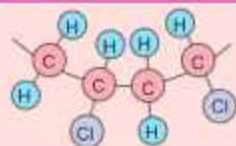


Amorphous solids

Polymers form a large group of naturally occurring and synthetic materials. Plastics and synthetic rubbers are termed as polymers because they are formed by polymerization reactions in which relatively simple molecules are chemically combined into massive long chain molecules, or three-dimensional structures. These materials have rather low specific gravity compared with even the lightest of metals, and yet exhibit good strength to weight ratio.

Polymers consist wholly or in part of chemical combinations of carbon with oxygen, hydrogen, nitrogen and other metallic or non metallic elements. Polythene, polystyrene and nylon, etc. are examples of polymers. Natural rubber is composed in the pure state entirely of a hydrocarbon with the formula $(C_5H_8)_n$.

For your information



Parts of a PVC molecule

5.2 MECHANICAL PROPERTIES OF SOLIDS

Deformation in Solids

If we hold a soft rubber ball in our hand and then squeeze it, the shape or volume of the ball will change. However, if we stop squeezing the ball, and open our hand, the ball will return to its original spherical shape. This has been illustrated schematically in Fig. 5.1.

Similarly, if we hold two ends of a rubber string in our hands, and move our hands apart to some extent, the length of the string will increase under the action of the applied force exerted by our hands. Greater the applied force, larger will be the increase in length. Now on removing the applied force, the string will return to its original length. From these examples, it is concluded that deformation (i.e., change in shape, length or volume) is produced when a body is subjected to some external force.

In crystalline solids, atoms are usually arranged in a certain order. These atoms are held about their equilibrium position, which depends on the strength of the inter-atomic cohesive force between them. Under the influence of external force, distortion occurs in the solid bodies because of the displacement of the atoms from their equilibrium position and the body is said to be in a state of stress. After the removal of external force, the atoms return to their equilibrium position, and the body regains its original shape, provided that external applied force was not too great. The ability of the body to return to its original shape is called elasticity. Figure 5.2 illustrates deformation produced in a unit cell of a crystal subjected to an external applied force.

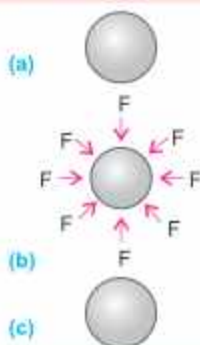


Fig. 5.1:

- (a) Original rubber ball
- (b) Squeezed rubber ball subjected force F by the hand
- (c) Rubber ball after removing force



(a) Unstretched unit cell



(b) Unit cell under outward stretching force



(c) Unit cell under inward applied force



(d) Unit cell after removing applied force

Fig. 5.2

5.3 STRESS, STRAIN AND YOUNG'S MODULUS

The results of mechanical tests are usually expressed in terms of stress and strain, which are defined in terms of applied force and deformation.

Stress

It is defined as the force applied per unit area to produce any change in the shape, volume or length of a body. Mathematically, it is expressed as:

$$\text{Stress } (\sigma) = \frac{\text{Force } (F)}{\text{Area } (A)} \quad \dots\dots\dots (5.1)$$

The SI unit of stress (σ) is newton per square metre (N m^{-2}), which is given the name pascal (Pa). Stress may cause a change in length, volume and shape. When a stress changes length, it is called the tensile stress, when it changes the volume, it is called the volume stress and when it changes the shape, it is called the shear stress.

Strain

Strain is a measure of the deformation of a solid when stress is applied to it. In the case of deformation in one dimension, strain is defined as the fractional change in length. If ΔL is the change in length and L_0 is the original length (Fig. 5.3-a), then strain is given by

$$\text{Strain } (\varepsilon) = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L_0)} \quad \dots\dots\dots (5.2)$$

Since strain is the ratio of lengths, it is dimensionless and therefore, has no units. If strain ε is due to tensile stress σ , it is called tensile strain, and if it is produced as a result of compressive stress σ , it is termed as compressive strain.

In case when the applied stress changes the volume, the change in volume per unit volume is known as volumetric strain as shown in Fig. 5.3 (b), thus

$$\text{Volumetric strain } (\varepsilon_v) = \frac{\Delta V}{V_0} \quad \dots\dots\dots (5.3)$$

Let y be the distance between two opposite faces of a rigid body (Fig. 5.3-c), which are subjected to shear stress one of its face slides through a distance Δx , then shear strain is produced which is given by

$$\text{Shear strain } (\gamma) = \frac{\Delta x}{y} = \tan \theta \quad \dots\dots\dots (5.4)$$

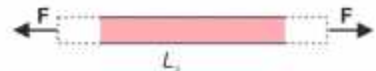


Fig. 5.3(a): Tensile strain

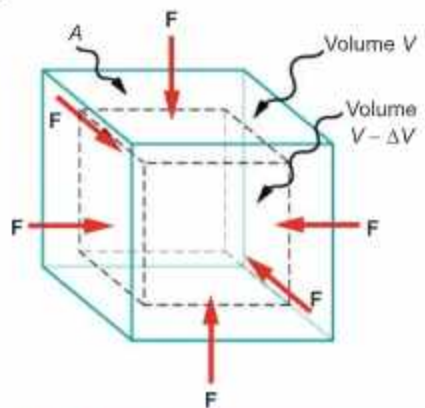


Fig. 5.3(b): Volumetric strain

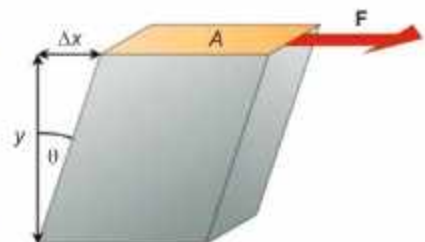


Fig. 5.3(c): Shear strain

However, for small value of angle θ , measured in radian $\tan\theta \cong \theta$, so that

$$\gamma = \theta \quad \dots\dots\dots(5.5)$$

Young's Modulus

The stress applied per unit strain is called Young's modulus

i.e.
$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$Y = \frac{F/A}{\Delta L/L_0} \quad \dots\dots\dots (5.6)$$

It has the same unit as that of stress i.e., N m^{-2} . The value of Young's modulus of different material is given in Table 5.1.

There are various methods to determine the Young's modulus of a wire. One of the method is Searle's method.

For your information

Although it is named after the 19th century British Scientist Thomas Young, the concept was developed in 1727 by Leonhard Euler.

Table 5.1: Elastic constants for some materials

Material	Young's Modulus 10^9 N m^{-2}	Bulk Modulus 10^9 N m^{-2}	Shear Modulus 10^9 N m^{-2}
Aluminium	70	70	30
Bone	15	-	80
Brass	91	61	36
Concrete	25	-	-
Copper	110	140	44
Diamond	1120	540	450
Glass	55	31	23
Ice	14	8	3
Lead	15	7.7	5.6
Mercury	0	27	0
Steel	200	160	84
Tungsten	390	200	150
Water	0	2.2	0

5.4 DETERMINATION OF YOUNG'S MODULUS OF A WIRE

Experimentally, the magnitude of Young's modulus for a material in the form of wire can be found out mostly with the help of Searle's apparatus as shown in Fig. 5.4.

It consists of two wires, reference wire and test wire of equal lengths of same material having same diameters attached to a rigid support. Both wires are connected to horizontal bars (frames F_1 and F_2) at the other ends. Hang a constant weight to the hook of horizontal bar of reference wire and hanger on test wire so that wire remains stretched and free from kinks.

Procedure

The following procedure is adopted for finding Young's modulus of a wire experimentally.

1. Measure the initial length L_0 of the wire using a metre scale.
2. Measure the diameter ' d ' of the wire using a screw gauge. The diameter should be measured at several

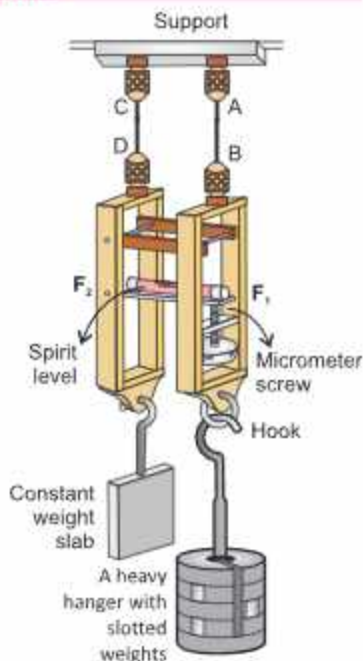


Fig. 5.4: Searle's apparatus

different points along the wire and take average.

- Adjust the spirit level so that it is in horizontal position by turning the micrometer. Record the micrometer reading to use it as the reference reading.
- Load the test wire with a further weight, the spirit level tilts due to elongation of the test wire.
- Adjust the micrometer screw to restore the spirit level in the horizontal position. Subtract the first micrometer reading from the second micrometer reading to obtain the extension of the test wire.
- Calculate stress and strain from the following formula:

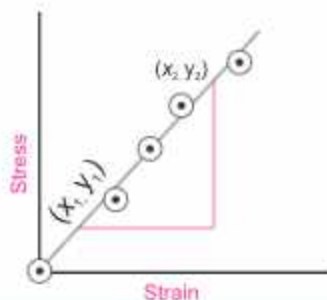
$$\text{Stress} = \frac{\text{Weight}}{\text{Area of wire}} = \frac{F}{A} = \frac{mg}{\pi r^2}$$

$$\text{Strain} = \frac{\Delta L}{L_0} = \frac{\text{Change in length}}{\text{Original length}}$$

- Repeat the above steps by increasing load on test wire to obtain more values of stresses and strains.
- Plot the above values on stress strain graph, it should be straight line. Now determine the value of slope Y . The value of slope is equal to Young's modulus of wire.

Brain teaser

A steel rod and a rubber band are subjected to a same force. Which one will be stretched more?



5.5 ELASTIC DEFORMATION, PLASTIC DEFORMATION AND ELASTIC LIMIT

In a tensile test machine, metal wire is extended at a specified deformation rate, and stresses generated in the wire during deformation are continuously measured by a suitable electronic device fitted in the mechanical testing machine. Force-elongation diagram or stress-strain curve is plotted automatically on X-Y chart recorder. A typical stress-strain curve for a ductile material is shown in Fig. 5.5.

In the initial stage of deformation, stress is increased linearly with the strain till we reach point A on the stress-strain curve. This is called proportional limit (σ_p). It is defined as the greatest stress that a material can withstand without losing straight line proportionality between stress and strain. Hooke's law which states that the strain (deformation) is directly proportional to stress (force or load) is obeyed in the region OA. From A to B, stress and strain are not proportional.

Elastic limit

If the load is removed at any point between O and B, the curve will be retraced and the material will return to its original state. In the region OB, the material is said to be

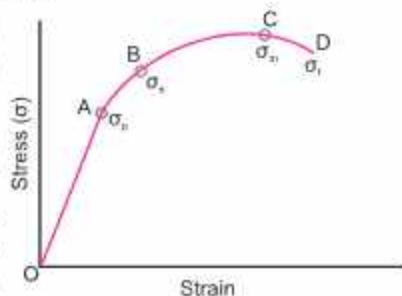


Fig. 5.5: Stress-strain curve of a typical ductile material.

elastic. The point B is called the yield point. The value of stress at B is known as elastic limit σ_e .

Plastic Deformation

If the stress is increased beyond the yield stress or elastic limit of the material, the specimen becomes permanently changed and does not recover its original shape or dimension after the stress is removed. This kind of behaviour is called plasticity. The region of plasticity is represented by the portion of the curve from B to C, the point C in Fig. 5.5 represents the ultimate tensile strength (UTS) σ_m of the material. The UTS is defined as the maximum stress that a material can withstand, and can be regarded as the nominal strength of the material. Once point C corresponding to UTS is crossed, the material breaks at point D, responding the fracture stress (σ_f).

Ductile substances

Substances which undergo plastic deformation until they break, are known as ductile substances. For example, Lead, copper and wrought iron are ductile substances.

Brittle Substances

The substances which break just after the elastic limit is reached, are known as brittle substances. For example, glass and high carbon steel are brittle. Moreover, Beryllium, Bismuth, Chromium are also brittle metals.

Example 5.1 A steel wire 12 mm in diameter is fastened to a log and is then pulled by a tractor. The length of steel wire between the log and the tractor is 11 m. A force of 10,000 N is required to pull the log. Calculate: (a) the stress and strain in the wire. (b) how much does the wire stretch when the log is pulled? ($E = 200 \times 10^9 \text{ N m}^{-2}$)

Solution (a) Tensile stress $\sigma = \frac{F}{A} = \frac{10,000 \text{ N}}{3.14(6 \times 10^{-3} \text{ m})^2} = 88.46 \times 10^6 \text{ N m}^{-2}$

Tensile strain $\epsilon = \frac{\Delta L}{L_0}$, also $E = \frac{\text{Stress}}{\text{Strain}} = \frac{88.46 \times 10^6 \text{ N m}^{-2}}{\text{Strain}} = 200 \times 10^9 \text{ N m}^{-2}$

$$\text{Strain} = \frac{88.46 \times 10^6 \text{ N m}^{-2}}{200 \times 10^9 \text{ N m}^{-2}} = 4.4 \times 10^{-4}$$

(b) Using the relation; Strain = $\frac{\Delta L}{L_0}$ or $\Delta L = \text{Strain} \times L_0 = 4.4 \times 10^{-4} \times 11 \text{ m} = 4.84 \times 10^{-3} \text{ m}$

5.6 STRAIN ENERGY IN DEFORMED MATERIALS

When a body is deformed by a force, work is done against elastic restoring force. It is stored in it as its potential energy and is equal to the gain in potential energy of the molecules of a body due to the displacement of these molecules from their mean positions.

Derivation of Expression for Energy Stored in a Stretched Material

Consider a material in the form of a spring as shown in Fig. 5.6. It is stretched by a

force F through extension x . As the extension is directly proportional to the stretching force within the elastic limit, therefore the force increases uniformly from zero to F as shown in Fig. 5.7. Thus, the average force that stretches the spring through x is $1/2F$. Hence work done by the stretching force will be given as:

Work done = Average force \times Distance in the direction of the force

$$W = \frac{1}{2} F \times x \quad \dots\dots\dots (5.7)$$

From Hooke's law $F = k(x)$

Therefore,
$$W = \left(\frac{1}{2}kx\right) \cdot (x) = \frac{1}{2} kx^2$$

or
$$W = \text{Area of OPQ}$$

The work done by the stretching force is stored in the spring as its strained energy and is equal to the potential energy stored in its molecules.

Strained energy stored in the body
$$E = \frac{1}{2} F \cdot x = \frac{1}{2} kx^2 \dots\dots\dots (5.8)$$

For your information

The amount of work done in stretching a material is equal to the average force applied multiplied by the distance moved. Therefore, the area under a force-extension graph represents the work done to stretch the material. Work done to stretch the material is also equal to elastic P.E. stored in the material.

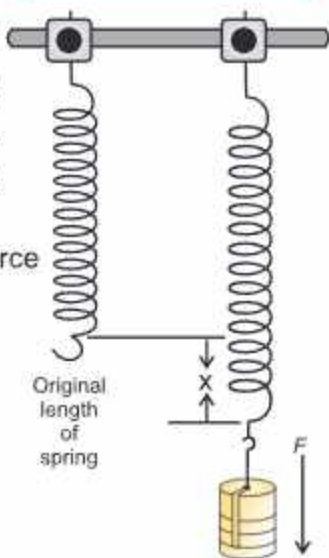


Fig. 5.6

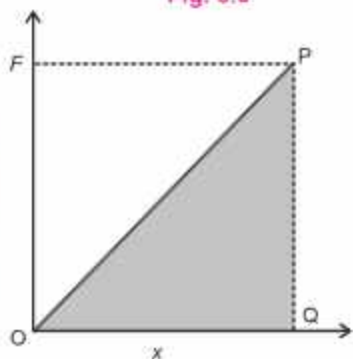


Fig. 5.7

5.7 ARCHIMEDES' PRINCIPLE AND FLOATATION

An air-filled balloon immediately shoots up to the surface when released under the surface of water. The same would happen if a piece of wood is released under water. We might have noticed that a mug filled with water feels light under water but feels heavy as soon as we take it out of water.

More than two thousand years ago, the Greek scientist, Archimedes noticed that there is an upward force which acts on an object which is kept inside a liquid. As a result, an apparent reduction in weight of the object is observed. This upward force acting on the object is called the upthrust of the liquid. Archimedes' principle states that:

When an object is totally or partially immersed in a liquid, an upthrust acts on it equal to the weight of the fluid it displaces.

Consider a solid cylinder of cross-sectional area A and height h immersed in a liquid as shown in Fig. 5.8. Let h_1 and h_2 be the depths of the top and bottom faces of the cylinder respectively from the surface of the liquid. Then

$$h_2 - h_1 = h \dots\dots\dots(5.9)$$

If P_1 and P_2 are the liquid pressures at depths h_1 and h_2 respectively and ρ is its density, then using equation $P = \rho gh$ of liquid pressure at height h :

$$P_1 = \rho gh_1$$

and $P_2 = \rho gh_2$

Let the force F_1 be exerted at the top of cylinder by the liquid due to pressure P_1 and the force F_2 be exerted at the bottom of the cylinder by the liquid due to P_2 ,

Then $F_1 = P_1 A = \rho gh_1 A$

and $F_2 = P_2 A = \rho gh_2 A$

F_1 and F_2 are the forces acting on the opposite faces of the cylinder. Therefore, the net force F will be equal to the difference of these forces. This net force F on the cylinder is called the upthrust of the liquid. Hence

$$F_2 - F_1 = \rho gh_2 A - \rho gh_1 A$$

$$= \rho g A (h_2 - h_1) \dots\dots\dots(5.10)$$

or Upthrust of liquid = $\rho g Ah$

$$\text{Upthrust} = \rho g V \dots\dots\dots(5.11)$$

Here Ah is the volume V of the cylinder and is equal to the volume of the liquid displaced by the cylinder, therefore, $\rho g V$ is the weight of the liquid displaced. This equation shows that an upthrust acts on a body immersed in a liquid and is equal to the weight of liquid displaced, which is Archimede's principle.

Example 5.2 A wooden cube of sides 10 cm each has been dipped completely in water. Calculate the upthrust of water acting on it.

Solution **Given**

Length of side $L = 10 \text{ cm} = 0.1 \text{ m}$

Volume $V = L^3 = (0.1 \text{ m})^3 = 1 \times 10^{-3} \text{ m}^3$

Density of water $\rho = 1000 \text{ kg m}^{-3}$

Upthrust $F = ?$

Using Archimede's principle

Upthrust of water = $\rho g V$

$$= 1000 \text{ kg m}^{-3} \times 9.8 \text{ m s}^{-2} \times 1 \times 10^{-3} \text{ m}^3 = 9.8 \text{ N}$$

Thus, upthrust of water acting on the wooden cube is 9.8 N.

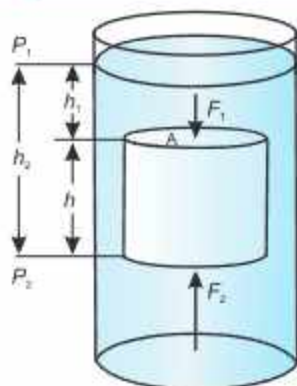


Fig. 5.8: Upthrust on a body immersed in a liquid is equal to the weight of the liquid displaced.

For your information

Archimedes was born about 287 BC, in Syracuse on the Island of Sicily. He was killed by a Roman soldier after he refused to leave his mathematical work.

Brain teaser

Why does a ship made of heavy steel float on water, while a small rock sink?

Floatation

An object sinks into a fluid if its weight is greater than the upthrust acting on it. However, an object floats if its weight is equal or less than the upthrust. When an object floats in a fluid, the upthrust acting on it is equal to the weight of the object. In case of floating object, the object may be partially immersed. The upthrust is always equal to the weight of the fluid displaced by the object. This is the principle of floatation. It states that:

A floating object displaces a fluid having weight equal to the weight of the object.

Archimedes' principle is applicable on liquids as well as on gases. We find numerous applications of this principle in our daily life.

Applications

Following are some important applications of Archimedes' principle.

1. Hot-air balloon

The reason why hot-air balloons (Fig. 5.9) rise and float in mid-air is because of the density of the hot-air balloon is less than the surrounding air. When the upthrust of the surrounded air is more, it starts to rise. This is done by varying the quantity of hot air in the balloon.



Fig. 5.9

2. Wooden block floating on water

A wooden block floats on water. It is because the weight of an equal volume of water is greater than the weight of the block. According to the principle of floatation, a body floats if its displaced water is equal to the weight of the body when it is partially or completely immersed in water.

3. Ships and boats

Ships and boats are designed on the same principle of floatation. They carry passengers and goods over water. It would sink in water if its total weight becomes greater than the upthrust of water.



Fig.5.10 (a): A ship floating over water

4. Submarine

A submarine can travel over as well as under water using the same principle of floatation.

It floats over water when the weight of water equal to its volume is greater than its weight. Under this condition, it is similar to a ship and remains partially above water level. It has a system of tanks which can be filled with and emptied from seawater. When these tanks are



Fig.5.10 (b): Submarine

filled with seawater, the weight of the submarine increases. As soon as its weight becomes greater than the upthrust, it dives into water and remains under water. To come up on the surface, the tanks are made empty from seawater.

Example 5.3 An empty meteorological balloon weighs 80 N. It is filled with 10 cubic metres of hydrogen. How much maximum contents the balloon can lift besides its own weight? The density of hydrogen is 0.09 kg m^{-3} and the density of air is 1.3 kg m^{-3} .

Solution **Given:**

$$\begin{aligned} \text{Weight of the balloon } w &= 80 \text{ N} \\ \text{Volume of hydrogen } V &= 10 \text{ m}^3 \\ \text{Density of hydrogen } \rho_1 &= 0.09 \text{ kg m}^{-3} \\ \text{Density of air } \rho_2 &= 1.3 \text{ kg m}^{-3} \\ \text{Weight of hydrogen } w_1 &= ? \\ \text{Weight of the contents } w_2 &= ? \\ \text{Upthrust } F &= \text{Weight of air displaced} \\ &= \rho_2 g V \\ &= 1.3 \text{ kg m}^{-3} \times 9.8 \text{ m s}^{-2} \times 10 \text{ m}^3 \\ &= 127.4 \text{ N} \\ \text{Weight of hydrogen } w_1 &= \rho_1 V g \\ &= 0.09 \text{ kg m}^{-3} \times 10 \text{ m}^3 \times 9.8 \text{ m s}^{-2} \\ &= 8.82 \text{ N} \\ \text{Total weight lifted } F &= w + w_1 + w_2 \end{aligned}$$

To lift the contents, the total weight of the balloon should not exceed F .

$$\begin{aligned} \text{Thus } w + w_1 + w_2 &= F \\ 80 \text{ N} + 8.82 \text{ N} + w_2 &= 127.4 \text{ N} \end{aligned}$$

$$\text{or } w_2 = 38.58 \text{ N}$$

Thus, the maximum weight of 38.58 N can be lifted by the balloon in addition to its own weight.

5.8 STEADY, NON-VISCOUS AND IDEAL FLUID

Moving fluids have great importance. In order to find the behaviour of the fluid in motion, we consider their flow through the pipes. When a fluid is in motion, its flow can take place in two ways, either streamline or turbulent.

Streamline or Laminar Flow

The flow is said to be streamline or laminar flow.

If every particle that passes a particular point, moves along exactly the same path, as followed by particles which passed that points earlier.

In a steady flow of a fluid, the motion of the particles is smooth and regular, as shown in Fig. 5.11. The smooth path followed by fluid particles in laminar flow is called a streamline. The streamline may be the straight or curved and tangent to any point gives the direction of flow of a fluid. The different streamlines cannot cross each other.

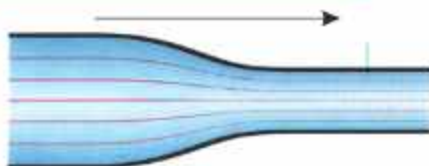


Fig. 5.11: Streamlines (laminar flow)

Example: A fluid flowing in a pipe as shown in Fig. 5.12 will have certain velocity v_1 at P, a velocity v_2 at Q and so on. If the velocity of a particle of the fluid at P, Q and R does not change with the passage of time, then the flow is said to be steady flow or streamline flow.

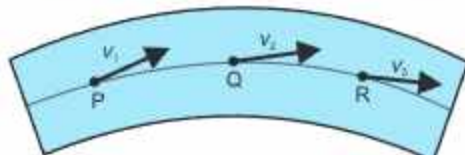


Fig. 5.12: The velocities of the particles at different points on streamline.

The line PQR which represents the path followed by the particle is called a streamline. It represents the fixed path followed by orderly processing particles. In streamline flow, all the particles passing through P also pass through Q and R. It means that two streamlines cannot cross each other.

Turbulent Flow

The irregular or unsteady flow of the fluid is called turbulent flow.

Above a certain velocity of the fluid flow, the motion of the fluid becomes unsteady and irregular. Under this condition, the velocity of the fluid changes abruptly as shown in the Fig. 5.13. In this case, the exact path of the particles cannot be considered.

If two streamlines cross each other, then the particles will go in one or in the other directions and flow will not be a steady flow. Such a flow is a turbulent flow. When the flow is unsteady or turbulent, there are eddies and whirlpools in the motion and the paths of the particles are continuously changing.

Ideal Fluid

The behaviour of the fluid which satisfies the following conditions is called ideal fluid:

1. The fluid is non-viscous i.e., there is no frictional force between adjacent layers of the fluid.



Fig. 5.13: Turbulent flow

For your information



Formula One racing cars have a streamlined design.



Dolphins have streamlined bodies to assist their movement in water.

- The fluid is incompressible i.e., its density is constant.
- The fluid motion is steady.

Rate of Flow

The rate of flow of a fluid through a pipe is the volume of the fluid passing through any section of pipe per unit time.

Formula For Rate of Flow

Consider a fluid flowing through a pipe of area of cross-section A as shown in Fig. 5.14. Let the velocity of the fluid be v and it flows through the pipe for time t , then the distance covered by the fluid in time is:

$$\ell = vt$$

where ℓ is the length of the pipe through which the fluid passes in time t . Volume of the fluid passing through the pipe in time t , is:

$$A \times \ell = Avt$$

Thus The rate of flow of the liquid = $\frac{\text{Volume}}{\text{Time}}$

$$= \frac{Avt}{t} = Av$$

$$\text{Rate of flow} = Av \dots\dots\dots(5.12)$$

In SI units, it is measured in cubic metre per second ($\text{m}^3 \text{s}^{-1}$). Sometimes, it is also measured in litres per second (L s^{-1}).

Steady Flow

If the overall flow pattern does not change with time, the flow is called steady flow.

In steady flow, every particle of the fluid follows the same flow line as its previous particle.

5.9 EQUATION OF CONTINUITY

Statement

The product of cross-sectional area of the pipe and the fluid speed (i.e., Av) at any point along the pipe is a constant. This constant is equal to the volume flow per second of the fluid or simply the flow rate.

Thus $Av = \text{Constant} = \frac{\text{Volume}}{\text{Time}}$

Consider a fluid flowing through a pipe of non-uniform size.

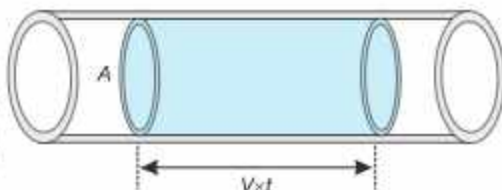


Fig. 5.14: Rate of flow of a liquid

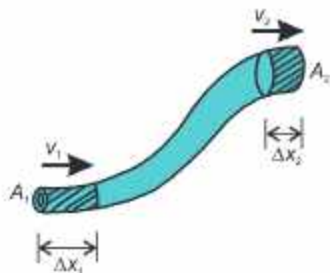


Fig. 5.15: Steady flow of a fluid

The particles in the fluid move along the same lines in a steady state flow as shown in Fig 5.15.

If we consider the flow for a short interval of time Δt , the fluid at the lower end of the tube covers a distance Δx , with a velocity v_1 , then distance covered by the fluid is:

$$\Delta x_1 = v_1 \Delta t \dots \dots \dots (5.13)$$

Let A_1 be the area of cross-section of the lower end, then volume of the fluid that flows into the tube at A_1 is:

$$V = A_1 \Delta x_1$$

or $V = A_1 v_1 \Delta t$

If ρ_1 is the density of the fluid, then the mass of the fluid contained in the shaded region (through A_1) is:

$$\Delta m_1 = \text{Volume} \times \text{Density}$$

or $\Delta m_1 = A_1 v_1 \Delta t \times \rho_1$

Similarly, the mass of the fluid that moves with velocity v_2 through the upper end of the pipe having cross-sectional area A_2 in the same time Δt is given by

$$\Delta m_2 = A_2 v_2 \Delta t \times \rho_2$$

where ρ_2 is the density of the fluid flowing out through A_2 and Δm_2 indicates small mass.

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved. That is the mass flowing into the bottom of the pipe through A_1 in a time Δt must be equal to the fluid flowing out through A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2 \dots \dots \dots (5.14)$$

So $A_1 v_1 \Delta t \times \rho_1 = A_2 v_2 \Delta t \times \rho_2$

or $A_1 v_1 \rho_1 = A_2 v_2 \rho_2 \dots \dots \dots (5.15)$

Equation (5.15) is called the equation of continuity. Since density is constant for the steady flow of incompressible fluid, therefore, the equation of continuity becomes:

$$A_1 v_1 = A_2 v_2 \dots \dots \dots (5.16)$$

Equation (5.16) states that in steady flow, the rate of flow of the fluid inward is equal to the rate of flow of the fluid outward.

This equation justifies the conservation of mass of the fluid which is flowing through a pipe.

Example 5.4 A water hose with an internal diameter of 20 mm at the outlet discharge 30 kg of water in 60 s. Calculate the water speed at the outlet. Assume the density of water is 1000 kg m^{-3} and its flow is steady.

Internal diameter of water hose $D = 20 \text{ mm} = 0.02 \text{ m}$

Solution Radius

$$r = \frac{D}{2} = \frac{0.02 \text{ m}}{2} = 0.01 \text{ m}$$

Scientific Fact

Euler obtained the continuity equation for an incompressible fluid with a large number of terms in 1752. Later, it was translated by C. Truesdell from English in 1954.

Mass of water	$m = 30 \text{ kg}$
Time taken	$t = 60 \text{ s}$
Density of water	$\rho = 1000 \text{ kg m}^{-3}$
Speed of water	$v = ?$
Mass flow per second m/t	$= 30 \text{ kg} / 60 \text{ s}$ $= 0.5 \text{ kg s}^{-1}$
Cross-sectional area A	$= \pi r^2$ $= 3.14 \times (0.01 \text{ m})^2$ $= 3.14 \times 10^{-4} \text{ m}^2$

From equation of continuity, the mass of water discharging per second through area A is:

$$\rho A v = \text{Mass / Second}$$

$$v = \frac{\text{Mass / Second}}{\rho A}$$

$$v = \frac{0.5 \text{ kg s}^{-1}}{1000 \text{ kg m}^{-3} \times 3.14 \times 10^{-4} \text{ m}^2}$$

$$= 1.6 \text{ m s}^{-1}$$

Tidbits



As the water falls, its speed increases and so its cross sectional area decreases as mandated by the continuity equation.

For your information

The equation of continuity is applied to:

- blood flow in arteries and veins
- water flow in rivers and pipes
- air flow in duct and ventilation systems.

5.10 INCREASE IN FLOW VELOCITY

We can increase the flow velocity of water in a rubber pipe by squeezing it. When we squeeze the rubber pipe, we decrease the cross-sectional area through which the water flows. According to the equation of continuity,

$$A_1 v_1 = A_2 v_2$$

where A is the cross-sectional area and v is the flow velocity. By decreasing the cross-sectional area ($A_2 < A_1$), the velocity of the water (v_2) must increase to maintain the same flow rate. Therefore, squeezing the rubber pipe increases the flow velocity of fluid.

5.11 BERNOULLI'S EQUATION

The sum of pressure, *K.E.* per unit volume and *P.E.* per unit volume of an ideal fluid throughout its steady flow remains constant.

As the fluid moves through a pipe of varying cross-section and height, the pressure will change along the pipe. Bernoulli's equation is the fundamental equation in fluid dynamics that relates pressure to fluid speed and height.



Fig. 5.16: The speed of water spraying from the end of a garden hose increases as the hose is squeezed with the thumb.

In deriving Bernoulli's equation, we assume that the fluid is incompressible, non-viscous and flows in a steady state manner. Let us consider the flow of the fluid through the pipe in time t , as shown in Fig. 5.17.

The force on the upper end of the fluid is P_1A_1 , where P_1 is the pressure and A_1 is the area of cross-section at the upper end. The work done on the fluid, by the fluid behind it, in moving it through a distance Δx_1 , will be:

$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1$$

Similarly, the work done on the fluid at the lower end is:

$$W_2 = -F_2\Delta x_2 = -P_2A_2\Delta x_2$$

where P_2 is the pressure, A_2 is the area of cross-section of lower end and Δx_2 is the distance moved by the fluid in same time interval t . The work W_2 is taken to be -ve as this work is done against the fluid force. The net work done will be:

$$W = W_1 + W_2$$

or
$$W = P_1A_1\Delta x_1 - P_2A_2\Delta x_2 \dots \dots \dots (5.17)$$

If v_1 and v_2 are the velocities at the upper and lower ends respectively, then

$$W = P_1A_1v_1t - P_2A_2v_2t$$

From equation of continuity;

$$A_1v_1 = A_2v_2$$

Hence $A_1v_1t = A_2v_2t = V$ (volume)

So
$$W = (P_1 - P_2)V \dots \dots \dots (5.18)$$

If m is the mass and ρ is the density, then $V = \frac{m}{\rho}$. So, Eq. (5.18) becomes:

$$W = (P_1 - P_2) \frac{m}{\rho} \dots \dots \dots (5.19)$$

A part of this work is utilized by the fluid in changing its K.E. and a part is used in changing its gravitational P.E.

$$\text{Change in K.E.} = \Delta K.E. = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \dots \dots \dots (5.20)$$

$$\text{Change in P.E.} = \Delta P.E. = mgh_2 - mgh_1 \dots \dots \dots (5.21)$$

where h_1 and h_2 are the heights of the upper and lower ends respectively.

Applying the law of conservation of energy to this volume of the fluid, we have

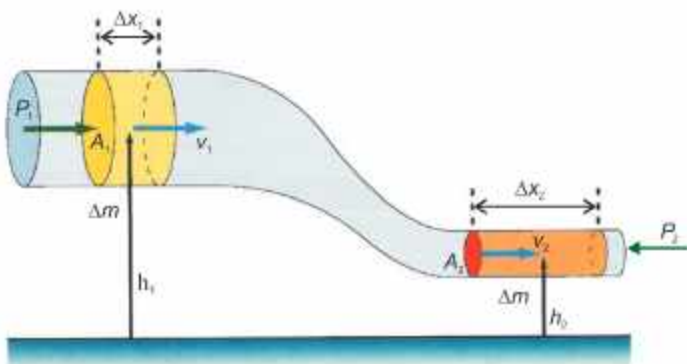


Fig. 5.17: An ideal flow of fluid through a non-uniform cross-section pipe at different heights.

Brain teaser

How does the shape of a curveball in baseball relate to Bernoulli's principle?

$$(P_1 - P_2) \frac{m}{\rho} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g h_2 - m g h_1, \dots\dots(5.22)$$

Rearranging Eq. (5.22), we have

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

This is Bernoulli's equation and is often expressed as:

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

5.12 USES OF BERNOULLI'S EQUATION

A number of devices operate by means of pressure difference that results from changes in the speed of the fluid.

1. Aeroplane Wings

Uplifting of an aeroplane is due to the designing of its wings, which deflect the air so that streamlines are closer together above the wing than below it as illustrated in Fig.5.18. We have seen that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than

on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward and the lift of an aeroplane is due to this effect.



Fig. 5.18: Lift of an aeroplane

2. Swing of a Ball

When a ball is thrown or kicked with spin or the ball is made smoother on one side by the bowler and remains rough on the other side, the air moves faster over rough side and slows over the smoother (Fig. 5.19). According to Bernoulli's equation, the faster moving air creates lower pressure, while the slower moving air creates higher pressure, this pressure difference generates a sideways force, known as Magnus effect which causes the ball to curves in the air.

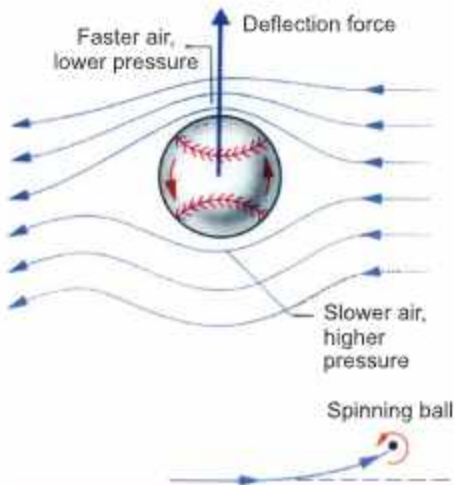


Fig. 5.19: Turbulent flow

3. Filter Pump

A filter pump has a constriction in the centre, so that a jet of water from the tap flows faster here. This causes a drop in pressure near it and air, therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump (Fig. 5.20).

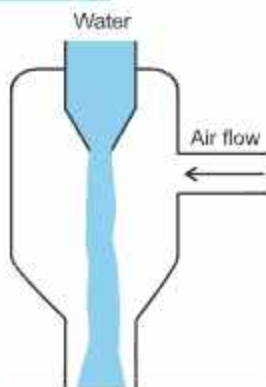


Fig. 5.20: Turbulent flow

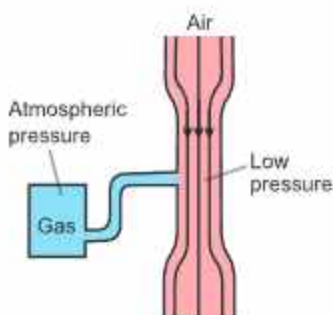


Fig. 5.21: Carburetor of an engine

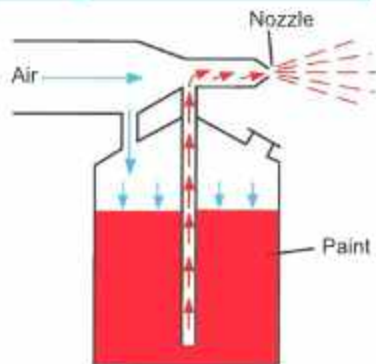


Fig. 5.22: A stream of air passing over a tube dipped in a liquid.

4. Carburetor

The carburetor of a car engine uses a Venturi duct to feed the correct mixture of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders (Fig. 5.21). A tiny inlet at the side of duct is fed with petrol.

The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapours into the air stream.

5. Paint Sprayer

A stream of air passing over a tube dipped in a liquid will cause the liquid to rise in the tube as shown in Fig. 5.22. This effect is used in perfume bottles and paint sprayers. Actually when the rubber ball of atomizer is squeezed, the air is blown through tube and it rushes out through the narrow aperture with high speed and it causes fall of pressure. So, the atmospheric pressure pushes the perfume up leading to the narrow aperture.

6. Venturi Relation

Consider a pipe within which a fluid of density ρ is flowing through different areas of cross-section as shown in the Fig. 5.23.

Let A_1 be the cross-sectional area at wide end and A_2 be the cross-sectional area at narrow portion.

Suppose that v_1 and v_2 be the flow speeds at the wide and narrow portions respectively. Pressure P_1 and P_2 indicate the liquid pressure at both the portions by connecting the limbs of the manometer.

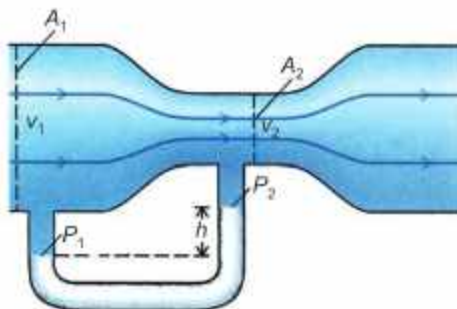


Fig. 5.23: Venturi meter

As the pipe is placed horizontally, therefore, we consider that average potential energy is the same at both places while using Bernoulli's equation.

Thus, Bernoulli's equation can be written as:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

or $P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$

or $P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$ (5.23)

From the equation of continuity:

$$A_1 v_1 = A_2 v_2$$

or $v_1 = \frac{A_2 v_2}{A_1}$

As the cross-sectional area A_2 is small as compared to the area A_1 , as is clear from the figure, i.e. $A_2 < A_1$. So, v_1 will be small as compared to v_2 . Thus, the speed of the fluid is very slow in wider portion of the pipe as compared to the narrow portion, So, we can neglect v_1 on the right-hand side of Eq.(5.23). Hence

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2$$
 (5.24)

This is known as Venturi relation, which is used in venturi meter, a device used to measure speed of liquid flow.

7. Torricelli's Theorem

A simple application of Bernoulli's equation is shown in Fig. 5.24. Suppose a large tank of fluid has two small orifices A and B on it. Let us find the speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds v_2 and v_3 will be much larger than the speed v_1 of the top surface of water. We can therefore, take v_1 as approximately zero. Hence, Bernoulli's equation can be written as:

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

But $P_1 = P_2 = \text{Atmospheric pressure}$

Therefore, the above equation becomes:

$$v_2 = \sqrt{2g(h_1 - h_2)}$$
.....(5.25)

This is Torricelli's theorem which states that:

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Interesting Information

It is clear from the result of Bernoulli's Equation for horizontal pipe that "where speed is high, the pressure will be low". Mathematically,

$$P + \frac{1}{2} \rho v^2 = \text{constant}$$

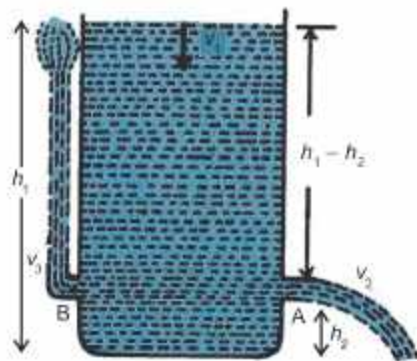


Fig.5.24:
A tank containing fluid with a orifice.

Notice that the speed of the efflux of liquid is the same as the speed of a ball that falls through a height $(h_1 - h_2)$. The top level of the tank has moved down a little and the *P.E.* has been transferred into *K.E.* of the efflux of fluid. If the orifice had been pointed upward at B as shown in Fig.6.4, this *K.E.* would allow the liquid to rise to the level of water tank. In practice, viscous-energy losses would alter the result to some extent.

5.13 VISCOUS DRAG AND STOKES' LAW

The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. Substances that do not flow easily, such as thick tar and honey, etc., have large coefficients of viscosity, usually denoted by Greek letter ' η '. Substances which flow easily, like water, have small coefficient of viscosities. Since liquids and gases have non zero viscosity, therefore, a force is required if an object is to be moved through them. Even the small viscosity of the air causes a large retarding force on a car as it travels at high speed. If you stick out your hand out of the window of a fast moving car, you can easily recognize that considerable force has to be exerted on your hand to move it through the air. These are typical examples of the following fact.

An object moving through a fluid experiences a retarding force called a drag force. The drag force increases as the speed of the object increases.

Even in the simplest cases, the exact value of the drag force is difficult to calculate. However, the case of a sphere moving through a fluid is of great importance.

The drag force F on a sphere of radius r moving slowly with speed v through a fluid of viscosity η is given by Stokes' law as under:

$$F = 6\pi\eta r v \dots\dots\dots (5.26)$$

However, at high speeds the force is no longer simply proportional to speed.

5.14 TERMINAL VELOCITY

Consider a water droplet having radius r such as that of fog falling vertically, the air drag on the water droplet increases with speed. The droplet accelerates rapidly under the over powering force of gravity which pulls the droplet rapidly downward due to force of gravity. However, the upward drag force on it increases as the speed of the droplet

For Your Information

Viscosities of Liquids and Gases at 30°C

Material	Viscosity $10^{-3} (\text{N s m}^{-2})$
Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.000
Plasma	1.6
Glycerin	6.29

Do you know?



A chimney works best when it is tall and exposed to air currents, which reduces the pressure at the top and forces the upward flow of smoke.

increases. The net force on the droplet is

$$\text{Net force} = \text{Weight} - \text{Drag force} \dots\dots\dots (5.27)$$

As the speed of the droplet continues to increase, the drag force eventually approaches the weight in the magnitude. Finally, when the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with constant speed called terminal velocity.

To find the terminal velocity v_t in this case, we use Stokes' law for the drag force. Equating it to the weight of the drop, we have

$$0 = mg - 6\pi\eta r v_t$$

$$v_t = \frac{mg}{6\pi\eta r} \dots\dots\dots (5.28)$$

The mass of the droplet is ρV , where $V = \frac{4}{3} \pi r^3$ is the volume of the sphere.

Substituting above values in the Eq. (5.28), we have

$$v_t = \frac{2gr^2\rho}{9\eta} \dots\dots\dots (5.29)$$

Example 5.5 A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that η for air = $19 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1}$ and density of water $\rho = 1000 \text{ kg m}^{-3}$.

Solution

$$r = 1.0 \times 10^{-4} \text{ m}, \quad \rho = 1000 \text{ kg m}^{-3}, \quad \eta = 19 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1}$$

Putting the above values in Eq. (5.29)

$$v_t = \frac{2 \times 9.8 \text{ m s}^{-2} \times (1.0 \times 10^{-4} \text{ m})^2 \times 1000 \text{ kg m}^{-3}}{9 \times 19 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1}}$$

Terminal velocity = 1.1 m s^{-1} .

5.15 REAL FLUIDS ARE VISCOUS FLUIDS

Ideal fluid

It is a fluid that does not have viscosity and cannot be compressed. This type of fluid cannot exist practically.

Real fluid

All types of fluids that possess viscosity are termed as real fluids.

Examples: Kerosene oil, castor oil and honey etc.

Comparison of Ideal and Real Fluids

An example of ideal fluid cannot be provided because it does not exist in the real world.

However, every fluid that we see around us like water, diesel, petrol, honey, etc. are real fluids. Moreover, differences in viscosity can be found in real life, for example, honey is more viscous than water. Bernoulli's equation states that the speed of fluid flow is increased as a result of a simultaneous decrease in the potential energy of the fluid or a decrease in the static pressure on the fluid. When a fluid is viscous, it essentially refers to the thickness of the fluid or the friction, the fluid faces while fluid flows. Therefore, ideal fluids do not face the opposing force and have a non-viscous flow, while real fluids have a viscous flow. Ideal fluids are incompressible. It is not also subjected to surface tension.

5.16 SUPERFLUIDS

Superfluidity is the characteristic property of fluids with zero viscosity i.e., flow is frictionless. A substance exhibiting this property is a superfluid. Superfluids flow without loss of kinetic energy. They can flow through incredibly narrow spaces without any friction.

Superfluidity is achieved in some substances at extremely low temperature. For example, in fluid dynamics, a vortex is region in a fluid in which the flow revolves around an axial line, which may be straight or curved. The vortices are generally created at a moving boundary due to frictionless conditions. Vortices move with the fluid and dispersed by the action of viscosity.

Superfluid helium-4 is the most studied example of superfluidity. It changes from a liquid to a superfluids just a few degree below its boiling point of -452°F (-269°C or 4 K). Superfluids helium-4 moving as a normal clear liquid, but it has no viscosity. This means that once it starts to flow, it keeps moving past any obstacles.

Superfluidity Applications

Currently, there are few practical uses for superfluids. Superfluid helium-4 serves as a coolant for high-field magnets. Both helium-3 and helium-4 are utilized in advanced particle detectors. Researching superfluidity also helps us learn more about superconductivity.

Liquid helium is recognized for its great thermal conductivity and is used in cryogenic applications, including cooling superconducting magnets, scientific research, and medical uses. Additionally, it is employed in industry for leak testing and in the production of electronic and optical products.

Tidbit

Parachutes increase air resistance (drag) by creating a large surface area, which counteracts the force of gravity. This slows down the persons fall, allowing them to land safely.

Tidbit

Superfluids can "climb" up walls and over edges of containers because they do not experience friction like normal fluids do.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

- 5.1 The region of stress-strain curve which obeys Hooke's law is:
(a) proportional limit (b) elastic region (c) plastic region (d) yield limit
- 5.2 Which of the following is more elastic?
(a) Rubber (b) Wood (c) Sponge (d) Steel
- 5.3 Which of the following is polymer solid?
(a) Wool (b) Glass (c) Sodium chloride (d) Copper
- 5.4 The effect of decrease of pressure with the increase in speed of a fluid in horizontal pipe is:
(a) Torricelli's effect (b) Bernoulli's effect (c) Venture's effect (d) Doppler's effect
- 5.5 The pressure will be low when speed of a fluid is:
(a) zero (b) high (c) low (d) constant
- 5.6 As per law of fluid friction for steady streamline flow, the friction:
(a) varies proportionally to velocity of fluid
(b) varies inversely proportional to pressure
(c) does not depend on pressure
(d) first increases then decreases
- 5.7 If a stone is submerged in water and it weighs less in water than in air, this phenomenon is due to:
(a) the reduction of mass in water (b) increase of density in water
(c) buoyant force acting upwards (d) the gravitational force acting upward
- 5.8 The principle of floatation is a direct application of:
(a) Pascal's law (b) Bernoulli's principle
(c) Archimedes' principle (d) Newton's third law
- 5.9 An ideal flow of any fluid must satisfy:
(a) Pascal law (b) Bernoulli's equation
(c) Continuity equation only (d) Both (b) and (c)
- 5.10 The lift force experienced by an aeroplane wings is primarily due to:
(a) viscosity of air (b) density of air
(c) pressure difference above and below the wing (d) gravitational force

- 5.11 In medical field, a venture mask, used to deliver a known oxygen concentration to patients operates is based on:
- (a) Newton's third law
(b) Archimedes' principle
(c) Pascal's law
(d) Bernoulli's principle
- 5.12 Which of the following is a defining characteristic of a superfluid?
- (a) Zero viscosity
(b) Infinite density
(c) Zero temperature
(d) Infinite thermal conductivity

Short Answer Questions

- 5.1 What is meant by (i) cohesive force (ii) viscosity?
- 5.2 Differentiate between streamline and turbulent flow of a fluid.
- 5.3 How does pressure changes with depth in fluids?
- 5.4 How is variation in pressure related to speed of a fluid?
- 5.5 How is the flow rate related to the cross-sectional area and velocity of the fluid?
- 5.6 How do you study the variation in velocity of a fluid at different points in a hose with varying diameter?
- 5.7 How does an object float or sink according to Archimedes Principle?
- 5.8 How does Archimedes reportedly discover the principle that bears his name?
- 5.9 Why standing near fast moving train is dangerous? Explain briefly.
- 5.10 What are some potential applications of superfluidity?
- 5.11 Differentiate between stress, strain and Young's modulus. Write down their SI units.

Constructed Response Questions

- 5.1 The ratio stress/strain remains constant for small deformation. What will be effect on this ratio when the deformation made is very large?
- 5.2 When pure water falls on a flat glass plate, it spreads on the plate while the mercury, when falls on the same plate gets converted into small globules. Why?
- 5.3 According to Bernoulli's theorem, the pressure of a fluid should remain uniform in a pipe of uniform radius. But actually, it goes on decreasing. Why is it so?
- 5.4 Why wings of an aeroplane are rounded outward while flattened inward?
- 5.5 What is the difference in real fluid, ideal fluid and superfluid? Which of these really exists in the world? Explain.
- 5.6 Why is the study of superfluids important for advancing our knowledge of low temperature physics?

Comprehensive Questions

- 5.1 Explain in detail the classification of solids with respect to atomic arrangements.
- 5.2 What is Archimedes' principle? Explain it in detail for finding upthrust.
- 5.3 Justify that mass remains conserved when a fluid flows through a pipe.
- 5.4 Explain the term superfluidity.
- 5.5 State and derive equation of continuity.
- 5.6 State and prove Bernoulli's equation.
- 5.7 Give some practical applications of Bernoulli's equation.
- 5.8 Define terminal velocity of a body and show that terminal velocity is directly proportional to the square of radius of the body.

Numerical Problems

- 5.1 A steel wire of length 2 metres and cross-sectional area of $2 \times 10^{-6} \text{ m}^2$ is stretched by a force of 400 N. If the Young's modulus of steel is $2 \times 10^{11} \text{ N m}^{-2}$, calculate the extension of the wire. (Ans: 0.002 m)
- 5.2 A spring with a spring constant 200 N m^{-1} is stretched by 0.5 m. Find the elastic P.E. stored in the spring. (Ans: 25 J)
- 5.3 A copper wire of length 3 metres and cross-sectional area of $1 \times 10^{-6} \text{ m}^2$ is subjected to a force of 500 N. Calculate the stress and strain produced in the wire. (Young's modulus of copper $Y = 1.1 \times 10^{11} \text{ N m}^{-2}$) (Ans: $5 \times 10^8 \text{ N m}^{-2}$, 0.00455)
- 5.4 A block of wood of mass 10 kg and density of 600 kg m^{-3} is floating in water. Calculate the buoyant force acting on the block. (Density of water = 1000 kg m^{-3} .) (Ans: 98 N)
- 5.5 Water flows through a pipe with a diameter of 0.05 m at a velocity of 2 m s^{-1} . If the pipe narrows to a diameter of 0.03 m, calculate the velocity of water at narrow section. (Ans: 5.56 m s^{-1})
- 5.6 Water flows through a horizontal pipe with a velocity of 3 m s^{-1} and pressure of 200,000 Pa at point 1. At the nozzle (point 2), the pressure decreases to atmospheric pressure 101,300 Pa and the velocity increases to 14 m s^{-1} . Calculate the velocity of the water exiting the nozzle. (Density of water = 1000 kg m^{-3}) (Ans: 14.37 m s^{-1})
- 5.7 A tank filled with water has a hole at a depth of 5 m from the water surface. Calculate the velocity of water flowing out of the hole. (Ans: 9.9 m s^{-1})
- 5.8 Calculate the terminal velocity of a spherical raindrop with a radius 0.5 mm falling through the air. (η for air = $19 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1}$, $\rho = 1000 \text{ kg m}^{-3}$ for water) (Ans: 28.65 m s^{-1})

Learning Objectives

After studying this chapter, the students will be able to:

- ◆ Describe the basic assumptions of the kinetic theory of gasses. [Including understanding the temperature, pressure and density conditions under which an ideal gas is a good approximation of a real gas.]
- ◆ State that regions of equal temperature are in thermal equilibrium
- ◆ Relate a rise in temperature of an object to an increase in its internal energy
- ◆ Apply the equation of state for an ideal gas [expressed as $PV = nRT$, where n = amount of substance (number of moles) and as $PV = Nk_B T$, where N = number of molecules]
- ◆ State that the Boltzmann constant k is given by $k_B = R/N_A$
- ◆ Use $W = P\Delta V$ for the work done when the volume of a gas changes at constant pressure.
- ◆ Describe the difference between the work done by a gas and the work done on a gas.
- ◆ Define and use the first law of thermodynamics [$\Delta U = Q - W$ expressed in terms of the increase in internal energy, the heating of the system (energy transferred to the system by heating) and the work done on the system]
- ◆ Explain qualitatively, in terms of particles, the relationship between the pressure, temperature and volume of a gas [Specifically the below case:
 - (a) pressure and temperature at constant volume
 - (b) volume and temperature at constant pressure
 - (c) pressure and volume at a constant temperature
- ◆ Use the equation, including a graphical representation of the relationship between pressure and volume for a gas at constant temperature.
- ◆ Justify how the first law of thermodynamics expresses the conservation of energy.
- ◆ Relate a rise in temperature of a body to an increase in its internal energy.
- ◆ State the working principle of a heat engine.
- ◆ Describe the concept of reversible and irreversible processes.
- ◆ State and explain the second law of thermodynamics.
- ◆ State the working principle of Carnot's engine
- ◆ Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine.
- ◆ Explain that an increase in temperature, increases the disorder of the system.
- ◆ Explain that increase in entropy means degradation of energy.
- ◆ Explain that energy is degraded during all natural processes.
- ◆ Identifying that system tends to become less orderly over time.
- ◆ Explain that Entropy, S is a thermodynamic quantity that relates to the degree of disorder of the particles in a system.
- ◆ State that the Carnot cycle sets a limit for the efficiency of a heat engine at the temperatures of its heat reservoir given by: Efficiency = $1 - \frac{T_{\text{cold reservoir}}}{T_{\text{hot reservoir}}}$

Thermodynamics is the branch of physics that deals with the relationships and conversions between heat and other forms of energy. It encompasses principles governing the behaviour of systems at macroscopic scales, such as temperature, pressure, and volume. Thermodynamics thus plays a key role in technology, since almost all the raw energy available for our use is liberated in the form of heat. In this chapter, we shall study the behaviour of gases and laws of thermodynamics, their significance and applications.

6.1 ASSUMPTIONS OF THE KINETIC THEORY OF GASES

The kinetic theory of gases is a fundamental theory in physics and chemistry that explains the behaviour of gases based on the motion of their constituent particles. This theory provides a macroscopic understanding of gas properties such as pressure, temperature, and volume. Here are the key assumptions of the kinetic theory of gases.

1. Gas Particles are in Constant, Random Motion

Gas molecules are in perpetual, random motion. They move in straight lines until they collide with either another molecule or the walls of the container.

2. Negligible Volume of Gas Particles

The volume of the individual gas molecules is negligible as compared to the total volume of the gas. This means that the particles are considered point masses with no significant volume.

3. No Intermolecular Forces

There are no attractive or repulsive forces between the gas molecules. The particles do not exert any force on each other except during collisions.

4. Elastic Collisions

The collisions between gas molecules, and with the walls of the container are perfectly elastic. This means that there is no net loss of kinetic energy during collisions. The total kinetic energy is conserved.

5. Large Number of Particles

A gas contains a large number of particles. This large number allows for the use of statistical methods to describe the properties of the gas.

6. Average Kinetic Energy is Proportional to Temperature

The average kinetic energy of gas particles is directly proportional to the absolute temperature of the gas. This implies that as the temperature increases, the speed of the gas particles also increases.

7. Pressure due to Particle Collisions

The pressure exerted by a gas on the walls of its container is due to the collisions of gas

particles with the walls. The force exerted by the particles during collisions generates pressure.

8. Time of Collisions is Negligible

The time interval of a collision between gas particles is extremely short compared to the time between collisions. This assumption simplifies the analysis of particle dynamics.

Limitations of Kinetic Molecular Theory

Real Gases

The assumptions of the kinetic theory hold true for ideal gases, but real gases exhibit deviations due to intermolecular forces and finite molecular volume, especially at high pressures and low temperatures.

In summary, the kinetic theory of gases provides a macroscopic view of gas behaviour, linking macroscopic properties like pressure and temperature to the motion of gas particles, and serves as a foundational concept in understanding thermodynamics and statistical mechanics.

Equation of State for an Ideal Gas

A gas that obeys kinetic theory of gases is termed as an ideal gas. Ideal gas equation is given by

$$PV = nRT \quad \dots\dots (6.1)$$

Here P represents pressure, V is volume, n is number of moles of the gas, R is universal gas constant ($R = 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}$) and T is the absolute temperature.

Equation (6.1) implies that product of pressure and volume is directly proportional to the absolute temperature for an ideal gas.

Real Gas to Behave Like an Ideal Gas

According to kinetic theory of gases, a gas has no intermolecular interaction and molecules are far apart from each other. For a real gas to behave like an ideal gas, some conditions must be satisfied. $P.E.$ of the gas molecules is negligible and this have only $K.E.$

Number of moles ' n ' can be given by

$$n = \frac{\text{Mass of gas}}{\text{Molar mass of gas}} = \frac{m}{M}$$

So, Eq. (6.1) becomes: $PV = \frac{m}{M}RT$ or $PM = \left(\frac{m}{V}\right)RT$

We know that density; $\rho = \frac{m}{V}$, So, $\rho = \frac{PM}{RT}$ as $\frac{M}{R}$ is constant $\therefore \rho \propto \frac{P}{T}$

The density of a gas will be low at low pressure and high temperature due to which molecules of the gas will be at large distance from each other and the intermolecular

forces will be negligible. So, the real gas behaves like an ideal gas at low pressure and high temperature.

Ideal Gas Equation in Terms of Boltzmann Constant

From ideal gas equation:

$$PV = nRT \dots\dots\dots (i)$$

A mole can be defined as the number of atoms or molecules per unit Avogadro's number ($N_A = 6.02 \times 10^{23}$).

Mathematically;

$$n = \frac{N}{N_A} \dots\dots\dots (ii) \quad , N \text{ is number of atoms or molecules}$$

Substituting Eq. (ii) in Eq. (i), we have

$$PV = \frac{N}{N_A} RT \dots\dots\dots (iii)$$

The term $\frac{R}{N_A}$ is termed as Boltzmann constant k_B , i.e.,

$$k_B = \frac{R}{N_A} \dots\dots\dots (6.2)$$

For your information

Real gases approach ideal behaviour under:

- (i) low pressure
- (ii) high temperature

Substituting the values of R and N_A , we have

$$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Substituting Eq. (6.2) in Eq. (iii),

$$PV = N k_B T \dots\dots\dots (6.3)$$

Equation (6.3) gives ideal gas equation in terms of Boltzmann constant k_B .

Example 6.1 One mole of an ideal gas is at a temperature of 300 K. If the Boltzmann constant is $1.38 \times 10^{-23} \text{ J K}^{-1}$, calculate the volume of the gas at a pressure of 1 atm. [1 atm = 101325 Pa]

Solution We know that:

$$PV = nRT$$

$$R = N_A \times k_B \quad \text{where} \quad k_B = \frac{R}{N_A}$$

$$\text{Here } V = \frac{nN_A k_B T}{P}$$

$$V = \frac{1 \text{ mol} \times 6.02 \times 10^{23} \text{ mol}^{-1} \times 1.38 \times 10^{-23} \text{ J K}^{-1} \times 300 \text{ K}}{101325 \text{ Pa}}$$

$$V \approx 0.0245 \text{ m}^3$$

Thus, volume of gas would be 0.0245 m^3 .

Gas Laws

There are some variables (state functions) which describe quantity of gas which includes pressure, volume, and temperature (P , V , and T) with change in one variable, the second variable changes while the third is kept constant. The laws that relate these variables mutually for an ideal gas are termed as gas laws.

Boyle's Law

This law was introduced by Robert Boyle in 1662, and it provides a relationship between pressure and volume of a gas at constant temperature. It states that for a fixed mass of an ideal gas, the pressure P exerted by a gas varies inversely with volume V occupied by the gas at constant temperature.

Mathematically;

$$P \propto \frac{1}{V} \text{ at constant } T$$

$$P = \text{constant} \frac{1}{V} \text{ or } PV = \text{constant}$$

$$\text{or } P_1V_1 = P_2V_2$$

Graphical representation of Boyle's law is shown in Fig. 6.1.

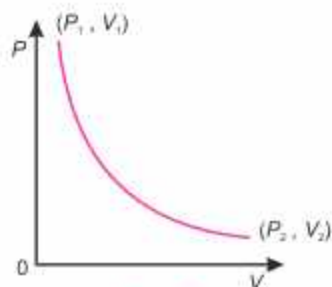


Fig. 6.1

Charles' Law

Charles' law relates volume and temperature of an ideal gas for a fixed mass at constant pressure. This law was formulated in 1870 by a French Physicist Jacques Charles. It states that the volume of given mass of gas at constant pressure is directly proportional to the absolute temperature.

Mathematically;

$$V \propto T \text{ at constant } P$$

$$\text{or } \frac{V}{T} = \text{constant}$$

$$\text{or } \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Graphically, it can be shown in Fig. 6.2.

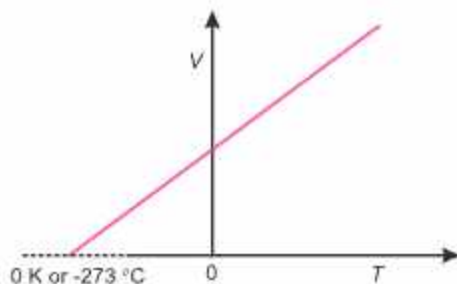


Fig. 6.2

Joseph Iussac's Law

It states that for a fixed mass of an ideal gas, the pressure exerted by a gas varies directly with absolute temperature of the gas at constant volume.

Mathematically;

$$P \propto T \text{ at constant } V$$

$$\text{or } P = \text{constant} \times T$$

$$\text{or} \quad \frac{P}{T} = \text{constant}$$

$$\text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Graphically, Joseph Lussac's law is shown in Fig. 6.3.

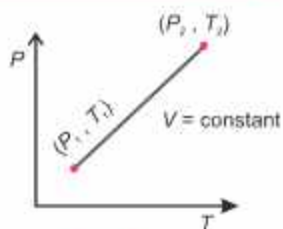


Fig. 6.3

Thermal Equilibrium

When two bodies are at the same temperature, the thermal energy (which is related to the kinetic energy of particles) of each body is equal. As a result, there is no driving force for heat transfer between them, and thus they remain in thermal equilibrium.

Example

When we put a metal spoon into a hot cup of coffee:

- (i) initially, the coffee is hotter than the spoon.
- (ii) over time, heat flows from the coffee to the spoon.
- (iii) eventually, the coffee and spoon reach at the same temperature.

Thermal equilibrium is achieved at this point, there is no net heat flow between the coffee and the spoon, and they are said to be in thermal equilibrium.

6.2 INTERNAL ENERGY

The sum of all forms of molecular energies (kinetic and potential) of a substance is termed as its internal energy. In the study of thermodynamics, usually ideal gas is considered as a working substance. The molecules of an ideal gas are mere mass points which exert no forces on one another. So, the internal energy of an ideal gas system is generally the translational *K.E.* of its molecules. Since the temperature of a system is defined as the average *K.E.* of its molecules, thus for an ideal gas system, the internal energy is directly proportional to its temperature.

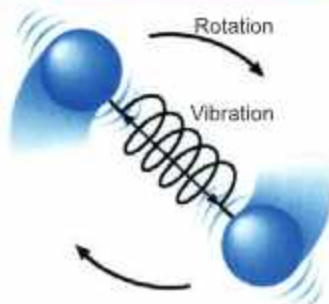
According to the kinetic theory of gases, the average kinetic energy of gas molecules is given by

$$\langle K.E. \rangle = \frac{3}{2} k_B T \text{ or } \langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T$$

where k_B is Boltzmann constant.

Therefore, the rise in temperature of an object reflects an increase in the internal kinetic energy of its particles. This increase in internal energy can

Do you know?



A diatomic gas molecule has both translational and rotational energy. It also has vibrational energy associated with the spring like bond between its atoms.

Tidbit

Different processes can lead to changes in internal energy and temperature, such as heating (adding heat), adiabatic compression or expansion (no heat exchange), or phase changes (where heat energy changes the state of matter without changing temperature).

occur due to the absorption of heat energy, which raises the average kinetic energy of the particles and thus increases the temperature of the object.

6.3 HEAT AND WORK

We know that both heat and work correspond to transfer of energy by some means. The idea was first applied to the steam engine where it was natural to transfer heat in and get work out. Consequently, it made a sense to define both heat in and work out as positive quantities. Hence, work done by the system on its environment is considered positive while work done on the system by the environment is taken as negative. If an amount of heat Q enters the system, it could manifest itself as either an increase in internal energy or as a resulting quantity of work performed by the system on the surrounding or both.

We can express the work in terms of directly measurable variables. Consider the gas enclosed in the cylinder with a moveable, frictionless piston of cross-sectional area A as shown in Fig. 6.4(a). In equilibrium, the system occupies volume V and exerts a pressure P on the walls of the cylinder and its piston. The force F exerted by the gas on the piston is PA .

We assume that the gas expands through ΔV very slowly, so that it remains in equilibrium as shown in Fig. 6.4(b). As the piston moves up through a small distance Δy , the work W done by the gas is:

$$W = F\Delta y = PA\Delta y$$

Since $A\Delta y = \Delta V$ (Change in volume)

$$\text{Hence } W = P\Delta V \quad \dots\dots\dots (6.4)$$

The work done can also be calculated by area of the curve under P - V graph as shown in Fig. 6.5.

Knowing the details of the change in internal energy and the mechanical work done, we are in a position to describe the general principles which deal with heat energy and its transformation into mechanical energy. These principles are known as laws of thermodynamics.

For your information

Internal energy is a function of state. Consequently, it does not depend on path but depends on initial and final states of the system. Thus, internal energy is similar to the gravitational P.E. So, like the potential energy, it is the change in internal energy and not its absolute value, which is important.

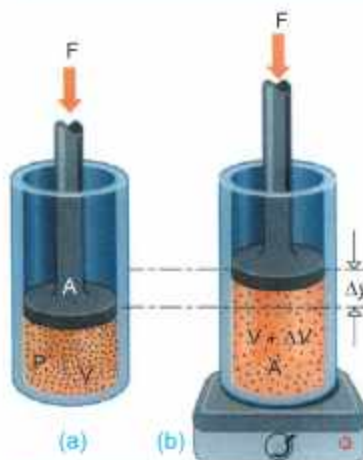


Fig. 6.4

A gas is sealed in a cylinder by a weightless, frictionless piston. The constant downward applied force F equals PA , and when the piston is displaced, downward work is done on the gas.

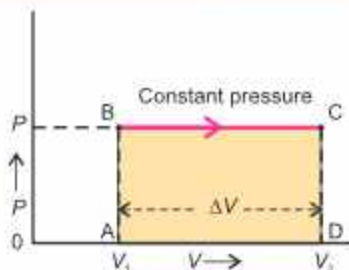


Fig. 6.5

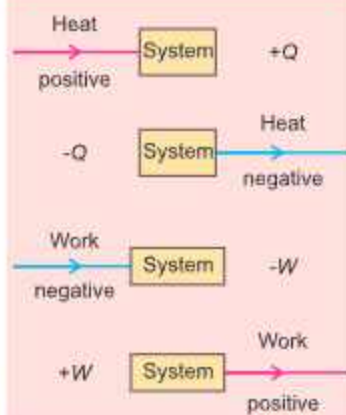
6.4 FIRST LAW OF THERMODYNAMICS

When heat is added to a system there is an increase in the internal energy due to the rise in temperature, an increase in pressure or change in the state. If at the same time, a substance is allowed to do work on its environment by expansion, the heat Q required will be the heat necessary to change the internal energy of the substance from U_1 in the first state to U_2 in the second state plus the work W done on the environment.

Thus $Q = (U_2 - U_1) + W$

or $Q = \Delta U + W$ (6.5)

For your information



Thus the change in internal energy $\Delta U = U_2 - U_1$ is defined as $Q - W$. Since it is the same for all processes concerning the state, the first law of thermodynamics, thus can be stated as,

In any thermodynamic process, when heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work W done by the system on its surroundings.

- Conservation Principle:** The underlying principle of the first law of thermodynamics is the conservation of energy. It asserts that while energy can change from one form to another (such as from chemical potential energy to thermal energy), the total energy in an isolated system remains constant over time.
- Wider Applicability:** Beyond mechanical systems, the first law of thermodynamics applies universally to all forms of energy and all types of processes, including chemical reactions, electrical systems, and nuclear reactions. It provides a foundational understanding that allows scientists and engineers to predict and understand energy transformations in various contexts.

The first law of thermodynamics expresses the law of conservation of energy by affirming that energy is a conserved quantity in isolated systems. It provides a framework to understand how energy is transferred and transformed within systems without violating the fundamental principle that energy neither be created nor be destroyed.

A bicycle pump is a good example of first law of thermodynamics. When we pump on the handle rapidly, it becomes hot due to mechanical work done on the gas, raising thereby its internal energy. One such simple arrangement is shown in Fig. 6.6. It consists of a bicycle pump with a blocked outlet.

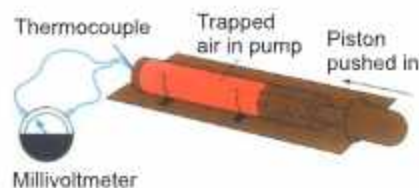


Fig. 6.6

A thermocouple connected through the blocked outlet allows the air temperature to be monitored. Thermocouple thermometer can detect a minute variation of the temperature. When the piston is rapidly pushed, thermometer shows a temperature rise due to increase of internal energy of the air. The push force does work on the air, thereby, increasing its internal energy, which is shown, by the increase in temperature of the air.

Human metabolism also provides an example of energy conservation. Human beings and other animals do work when they walk, run, or move. Work requires energy. Energy is also needed for growth to make new cells and to replace old cells that have died. Energy transforming processes that occur within an organism are named as metabolism. We can apply the first law of thermodynamics ($\Delta U = Q - W$), to an organism of the human body. Work done will result in the decrease in internal energy of the body. Consequently, the body temperature or in other words internal energy is maintained by the food we eat.

Example 6.2 A gas is enclosed in a container fitted with a piston of cross-sectional area 0.10 m^2 . The pressure of the gas is maintained at 8000 N m^{-2} . When heat is slowly transferred, the piston is pushed up through a distance of 4.0 cm . If 42 J heat is transferred to the system during the expansion, what is the change in internal energy of the system?

Solution The work done by the gas is

$$\begin{aligned} W &= P\Delta V = PA\Delta y = 8000 \text{ N m}^{-2} \times 0.10 \text{ m}^2 \times 4.0 \times 10^{-2} \text{ m} \\ &= 32 \text{ N m} = 32 \text{ J} \end{aligned}$$

The change in internal energy is found from first law of thermodynamics

$$\Delta U = Q - W = 42 \text{ J} - 32 \text{ J} = 10 \text{ J}$$

Isothermal Process

It is a process which is carried out at constant temperature and hence the condition for the application of Boyle's law on the gas is fulfilled. Therefore, when gas expands or compresses isothermally, the product of its pressure and volume during the process remains constant. If P_1, V_1 are initial pressure and volume whereas P_2, V_2 are pressure and volume after the isothermal change takes place (Fig. 6.7-a), respectively, then

$$P_1 V_1 = P_2 V_2$$

In case of an ideal gas, the P.E. associated with its molecules is zero, hence, the internal energy of an ideal gas depends only on its temperature, which in this case remains constant, therefore, $\Delta U = 0$. Hence, the first law of thermodynamics reduces to

$$Q = W$$

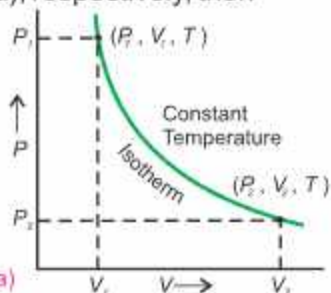


Fig. 6.7(a)

Thus if gas expands and does external work W , an amount of heat Q has to be supplied to the gas in order to produce an isothermal change. Since transfer of heat from one

place to another requires time, hence, to keep the temperature of the gas constant, the expansion or compression must take place slowly. The curve representing an isothermal process is called an isotherm (Fig. 6.7-a).

Adiabatic Process

An adiabatic process is the one in which no heat enters or leaves the system. Therefore, $\Delta Q = 0$ and the first law of thermodynamics gives:

$$W = -\Delta U$$

Thus, if the gas expands and does external work, it is done at the expense of the internal energy of its molecules and, hence, the temperature of the gas falls. Conversely, an adiabatic compression causes the temperature of the gas to rise because of the work done on the gas.

Adiabatic change occurs when the gas expands or compressed rapidly, particularly when the gas is contained in an insulated cylinder. The examples of adiabatic process are:

- The rapid escape of air from a burst tyre.
- The rapid expansion and compression of air through which a sound wave is passing.
- Cloud formation in the atmosphere.

As the temperature of the gas does not remain constant, so it has been seen that;

$$PV^\gamma = \text{constant}$$

where γ is the ratio of the molar specific heat of the gas at constant pressure to molar specific heat at constant volume ($\gamma = \frac{C_p}{C_v}$). The curve representing an adiabatic process is called an adiabat (Fig. 6.7-b).

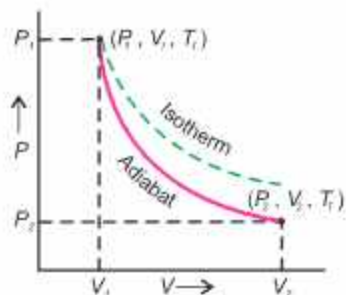


Fig. 6.7(b)

Brain teaser!

Why does the internal energy of an ideal gas remain constant during isothermal expansion?

6.5 REVERSIBLE AND IRREVERSIBLE PROCESSES

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings. In the reverse process, the working substance passes through the same stages as in the direct process but thermal and mechanical effects at each stage are exactly reversed. If heat is absorbed in the direct process, it will be given out in the reverse process and if work is done by the substance in the direct process, work will be done on the substance in the reverse process. Hence, the working substance is restored to its original conditions.

A succession of events which brings the system back to its initial condition is called a cycle. A reversible cycle is the one in which all the changes are reversible.

Although no actual change is completely reversible but the processes of liquification and evaporation of a substance, performed slowly, are practically reversible. Similarly the

slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by slowly decreasing the pressure on the piston to reverse the operation.

If a process cannot be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.

All changes which occur suddenly or which involve friction or dissipation of energy through conduction, convection or radiation are irreversible. An example of highly irreversible process is an explosion.

Do you know?



First practical steam-engine was designed by John Braithwaite and John Ericsson in England around 1829.

6.6 HEAT ENGINE

A heat engine converts some thermal energy to mechanical work. Usually, the heat comes from the burning of a fuel. The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off showing the ability to do work. This observation helped to develop a steam engine.

The working principle of a heat engine is based on the conversion of heat energy into mechanical work through a cyclic process. Here is how a heat engine typically operates. The working principle of a heat engine involves the cyclic transfer of heat energy from a high temperature reservoir to a low temperature reservoir, with the objective of converting as much heat as possible into mechanical work. This process is governed by principles of thermodynamics and is essential in various applications where mechanical energy is required from heat sources.

6.7 SECOND LAW OF THERMODYNAMICS

First law of thermodynamics tells us that heat energy can be converted into equivalent amount of work, but it is silent about the conditions under which this conversion takes place. The second law is concerned with the circumstances in which heat can be converted into work and direction of flow of heat.

Before initiating the discussion on formal statement of the second law of thermodynamics, let us analyze briefly the factual operation of an engine. The engine or the system (Fig. 6.8) absorbs a quantity of heat Q_1 from the heat source at temperature T_1 . It does work W and expels heat Q_2 to low temperature reservoir at temperature T_2 . As the

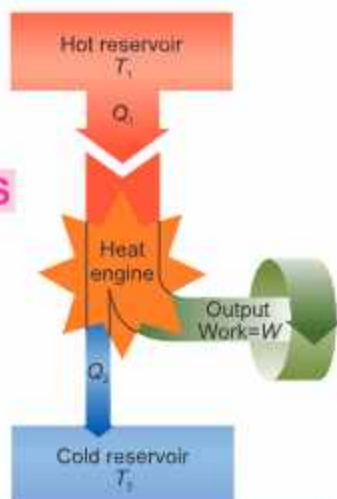


Fig. 6.8

Schematic representation of a heat engine. The engine absorbs heat Q_1 from the hot reservoir, expels heat Q_2 to the cold reservoir and does work W .

working substance goes through a cyclic process, in which the substance eventually returns to its initial state, the change in internal energy is zero. Hence, from the first law of thermodynamics, net work done should be equal to the net heat absorbed. i.e.,

$$W = Q_1 - Q_2$$

In practice, the petrol engine of a motor car extracts heat from the burning fuel and converts a part of this energy to mechanical energy or work and expels the rest to the atmosphere. It has been observed that petrol engines convert roughly 25% and diesel engines 35 to 40% available heat energy into work.

The second law of thermodynamics is a formal statement based on these observations. It can be stated in a number of different ways:

According to Lord Kelvin's statement based on the working of a heat engine:

It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

This means that a single heat reservoir, no matter how much energy it contains, cannot be made to perform any work. This is true for oceans and our atmosphere which contain a large amount of heat energy but cannot be converted into useful mechanical work. As a consequence of second law of thermodynamics, two bodies at different temperatures are essential for the conversion of heat into work. Hence, for the working of heat engine there must be a source of heat at a high temperature and a sink at low temperature to which heat may be expelled. The reason for our inability to utilize the heat contents of oceans and atmosphere is that there is no reservoir at a temperature lower than any one of the two.



6.8 CARNOT ENGINE AND CARNOT'S THEOREM

Sadi Carnot in 1824 described an ideal engine using only isothermal and adiabatic processes. He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures, would be the most efficient engine. A Carnot cycle using an ideal gas as the working substance is shown on PV diagram (Fig. 6.9). It consists of following four steps:

1. The gas is allowed to expand isothermally at temperature T_1 absorbing heat Q_1 from the hot

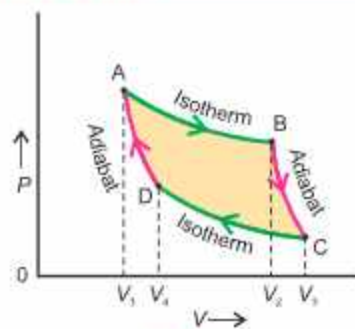


Fig. 6.9

reservoir. The process is represented by the curve AB.

- The gas is then allowed to expand adiabatically until its temperature drops to T_2 . The process is represented by the curve BC.
- The gas at this stage is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservoir. The process is represented by the curve CD.
- Finally the gas is compressed adiabatically to restore its initial state at temperature T_1 . The process is represented by the curve DA.

Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible. As the working substance returns to the initial state, there is no change in its internal energy i.e., $\Delta U = 0$.

The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram. It can also be estimated from net heat ΔQ absorbed in one cycle.

$$Q = Q_1 - Q_2$$

From 1st law of thermodynamics:

$$Q = \Delta U + W$$

or
$$W = Q_1 - Q_2 \quad (\because \Delta U = 0)$$

The efficiency η of the heat engine is defined as:

$$\eta = \frac{\text{Output (Work)}}{\text{Input (Energy)}}$$

Thus
$$\eta = \frac{Q_1 - Q_2}{Q_1} \dots\dots\dots (6.6)$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to kelvin temperature. So Q_1 and Q_2 are proportional to kelvin temperatures T_1 and T_2 respectively and hence,

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \dots\dots\dots (6.7)$$

The efficiency is usually taken in percentage, in that case:

$$\text{Percentage efficiency} = \left(1 - \frac{T_2}{T_1} \right) \times 100$$

Thus, the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. The larger the temperature difference of two reservoirs, the greater is the efficiency. But it can never be one or 100% unless cold reservoir is at absolute zero temperature ($T_2 = 0 \text{ K}$).

Such reservoirs are not available and hence the maximum efficiency is always less than

Interesting information



A waterfall analogy for the heat engine.

one. Nevertheless, the Carnot cycle establishes an upper limit on the efficiency of all heat engines. No practical heat engine can be perfectly reversible and also energy dissipation is inevitable. This fact is stated in Carnot's theorem:

No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

The Carnot's theorem can be extended to state that:

All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.

In most practical cases, the cold reservoir is near room temperature. So, the efficiency can only be increased by raising the temperature of hot reservoir. All real heat engines are less efficient than Carnot engine due to friction and other heat losses.

Example 6.3 The turbine in a steam power plant takes steam from a boiler at 427°C and exhausts into a low temperature reservoir at 77°C . What is the maximum possible efficiency?

Solution Maximum efficiency for any engine operating between temperatures T_1 and T_2 is

$$\% \eta = \frac{T_1 - T_2}{T_1} \times 100 \% \quad \text{where} \quad T_1 = 427 + 273 = 700 \text{ K}$$

and $T_2 = 77 + 273 = 350 \text{ K}$

$$\text{So} \quad \eta = \frac{T_1 - T_2}{T_1} = \frac{700 \text{ K} - 350 \text{ K}}{700 \text{ K}} = \frac{350 \text{ K}}{700 \text{ K}} = \frac{1}{2} = 0.5 \quad \text{or} \quad \% \eta = 0.5 \times 100 = 50 \%$$

6.9 REFRIGERATOR

Refrigerator is a device which maintains the temperature of a body below that of its surrounding. It operates in a cyclic process but in reverse as that of the heat engine as shown in Fig. 6.10. A refrigerator absorbs heat from a cold reservoir and gives it off to a hot reservoir. This shows that in a refrigerator, the work is done on the system while in a heat engine work is done by the system.

A refrigerator works on the basis of Clausius statement of second law of thermodynamics, i.e., a heat engine is operating in reverse. Heat Q_c is drawn from Low Temperature Reservoir (LTR) T_1 by compressor and is thrown into High Temperature Reservoir (HTR) T_2 with the help of external work done. The heat rejected to HTR (Q_h) is given by

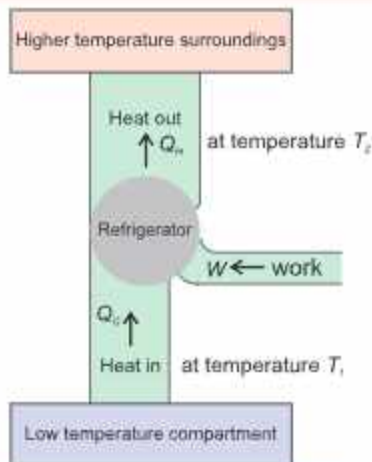


Fig. 6.10

A refrigerator transfers heat from a low-temperature compartment to higher-temperature surroundings with the help of external work. It is a heat engine operating in reverse order.

$$Q_c + W = Q_H \quad \text{or} \quad W = Q_H - Q_c$$

The main purpose of refrigerator is to extract as much heat Q_c as possible from LTR with the expenditure of as little work W as possible.

Coefficient of Performance of Refrigerator

The ratio of heat removed from LTR (Q_c) to the work done (W) is called coefficient of performance of a refrigerator.

A better refrigerator will remove a greater amount of heat from inside the refrigerator for the expenditure of a smaller mechanical work or electrical energy. The coefficient of performance of a refrigerator can be given by

$$E = \frac{Q_c}{W} = \frac{Q_c}{Q_H - Q_c}$$

The coefficient of performance in terms of temperature, where $Q \propto T$ is:

$$E = \frac{T_1}{T_2 - T_1} \quad \dots\dots\dots (6.8)$$

Example 6.4

A refrigerator has a coefficient of performance 8. If temperature in the freezer is -23°C , then what is the temperature at which it rejects the heat?

Solution

Coefficient of performance $E = 8$

Temperature of cold reservoir (freezer) $T_1 = -23^\circ\text{C} = -23 + 273 = 250 \text{ K}$

Temperature of hot reservoir (room) $T_2 = ?$

Coefficient of performance $= \frac{T_1}{T_2 - T_1}$

Substituting the values $8 = \frac{250 \text{ K}}{T_2 - 250 \text{ K}}$

or $8(T_2 - 250 \text{ K}) = 250 \text{ K}$ or $T_2 - 250 \text{ K} = \frac{250 \text{ K}}{8}$

or $T_2 = 31.25 \text{ K} + 250 \text{ K} = 281.25 \text{ K} = 8.25^\circ\text{C}$

6.10 ENTROPY

The concept of entropy was introduced into the study of thermodynamics by Rudolph Clausius in 1856 to give a quantitative basis for the second law. It provides another variable to describe the state of a system to go along with pressure, volume, temperature and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of heat ΔQ at absolute temperature T , then the increase in the state variable called entropy S of the system is given by

$$\Delta S = \frac{\Delta Q}{T} \dots\dots\dots (6.9)$$

Like potential energy or internal energy, it is the change in entropy of the system which is important.

The change in entropy is positive when heat is added and negative when heat is removed from the system. Suppose an amount of heat Q flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 when $T_1 > T_2$. The change in entropy of the reservoir, at temperature T_1 , which loses heat, decreases by Q/T_1 , and of the reservoir at temperature T_2 , which gains heat, increases by Q/T_2 . As $T_1 > T_2$, so Q/T_2 will be greater than Q/T_1 , i.e. $Q/T_2 > Q/T_1$.

Hence, net change in entropy ($\frac{Q}{T_2} - \frac{Q}{T_1}$) is positive.

It follows that in all natural processes where heat flows from one system to another, there is always a net increase in entropy. This is another statement of 2nd law of thermodynamics. It states that;

If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

It is observed that a natural process tends to proceed towards a state of greater disorder. Thus, there is a relation between entropy and molecular disorder. For example, an irreversible heat flow from a hot to a cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of increase in average molecular speeds and therefore, the randomness of molecular motion. Similarly, free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus, in both examples, entropy is said to be increased.

We can conclude that only those processes are probable for which entropy of the system increases or remains constant. The process for which entropy remains constant is a reversible process; whereas for all irreversible processes, entropy of the system increases.

Every time entropy increases, the opportunity to convert some heat into work is lost. For example, there is an increase in entropy when hot and cold waters are mixed. Finally, the warm water cannot be separated into a hot layer and a cold layer. There has been no loss of energy but some of the energy is no longer available for conversion into work. Therefore, increase in entropy means degradation of energy from a higher level where more work can be extracted to a lower level at which less or no useful work can be done. The energy in a sense is degraded, going from more orderly form to less orderly form, eventually ending up as thermal energy.

In all real processes where heat transfer occurs, the energy available for doing useful

work decreases, eventually, the entropy increases. Even if the temperature of some system decreases, thereby decreasing the entropy, it is at the expense of net increase in entropy for some other system. When all the systems are taken together as the universe, the entropy of the universe always increases.

Example 6.5 Calculate the entropy change when 1.0 kg ice at 0 °C melts into water at 0 °C. Latent heat of fusion of ice $L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$.

Solution

$$m = 1 \text{ kg}$$

$$T = 0^\circ\text{C} = 273 \text{ K}$$

$$L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$$

As $\Delta S = \frac{\Delta Q}{T}$

where $\Delta Q = mL_f$

So $\Delta S = \frac{mL_f}{T}$

Substituting the values

$$\Delta S = \frac{1.00 \text{ kg} \times 3.36 \times 10^5 \text{ J kg}^{-1}}{273 \text{ K}}$$

$$\Delta S = 1.23 \times 10^3 \text{ J K}^{-1}$$

Thus, entropy increases as it changes to water. The increase in entropy in this case is a measure of increase in the disorder of water molecules that change from solid to liquid state.

Brain teaser!

Why does a deck of cards become more disordered when shuffled?

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

- 6.1 In an isothermal change, internal energy:
 (a) decreases (b) increases (c) remains the same (d) becomes zero
- 6.2 First law of thermodynamics is based upon law of conservation of:
 (a) mass (b) energy (c) momentum (d) charge
- 6.3 A device which converts thermal energy into mechanical energy is called:
 (a) heat engine (b) Carnot engine (c) refrigerator (d) turbine
- 6.4 When two objects are made in thermal contact having same temperature, then they are at:
 (a) thermal Equilibrium (b) chemical Equilibrium
 (c) mechanical Equilibrium (d) physical Equilibrium

- 6.5 When the system is expanded by adding heat energy, then the work done will be:
(a) positive and on the system (b) negative and on the system
(c) positive and by the system (d) negative and by the system
- 6.6 Entropy of a system in reversible process:
(a) decreases (b) increases (c) is Infinite (d) is zero
- 6.7 What happens to internal energy of an object when its temperature:
(a) decreases (b) remains Constant (c) increases (d) fluctuates
- 6.8 The value of Boltzmann constant is:
(a) $1.38 \times 10^{-23} \text{ J K}^{-1}$ (b) $1.38 \times 10^{23} \text{ J K}^{-1}$
(c) $1.38 \times 10^{26} \text{ J K}^{-1}$ (d) $1.38 \times 10^{-23} \text{ J}^{-1} \text{ K}^{-1}$
- 6.9 In an adiabatic process, there is no:
(a) change in temperature (b) exchange of heat
(c) change in internal energy (d) work done
- 6.10 Thermodynamics mostly deals with:
(a) measurement of quantity of heat
(b) transfer of quantity of heat
(c) change of state
(d) Conversion of heat to other forms of energy

Short Answer Questions

- 6.1 What is meant by thermal equilibrium? Explain briefly.
- 6.2 What is meant by internal energy? How is it related to temperature of an ideal gas?
- 6.3 State 2nd law of thermodynamics in two different forms.
- 6.4 Is it possible to construct a heat engine of 100% efficiency? Explain.
- 6.5 Differentiate between reversible and irreversible processes.
- 6.6 Why adiabat is steeper than isotherm? Explain.
- 6.7 A refrigerator transforms heat from cold to hot body. Does this violate the second law of thermodynamics? Justify your answer.
- 6.8 Explain briefly heat death of universe in terms of entropy.
- 6.9 Is it possible for a cyclic reversible heat engine to absorb heat at constant temperature and transforms it completely into work without rejecting some heat at low temperature? Explain.
- 6.10 How does behaviour of real gases differ from ideal gas at high pressure and low temperature? Identify the reasons behind these differences based on kinetic theory of gases.
- 6.11 Show that area under P - V graph is equal to work done.
- 6.12 How is work done (i) by a gas (ii) on a gas?

Constructed Response Questions

- 6.1 Explain how thermodynamics relates to the concept of energy conservation.
- 6.2 Explain how thermodynamics applies to biological systems, such as human body.
- 6.3 A gas is expanding adiabatically. Explain what happens to temperature and pressure of the the gas.
- 6.4 A coffee cup is left on a table, and overtime coffee cup cools down. Explain thermodynamics processes occurring during this process.
- 6.5 How can we explain different weather patterns through thermodynamical processes like wind, rain, etc.

Comprehensive Questions

- 6.1 What are the postulates of kinetic theory of gases? Derive a relation for ideal gas equation in the form $PV=Nk_b T$ from general gas equation.
- 6.2 State and explain various gas laws.
- 6.3 Explain first law of thermodynamics in detail. Give an example in support of your explanation. Give its two applications.
- 6.4 What is a refrigerator? Explain its working. Derive an expression for its co-efficient of performance.
- 6.5. What is Carnot engine? Describe Carnot cycle. State Carnot theorem and derive an expression for efficiency of Carnot engine.
- 6.6 Define and explain the term "Entropy".

Numerical Problems

- 6.1 A gas occupies 6.0 L of volume at a pressure of 12 atm. What will be the volume of gas if the pressure is increased by 2.0 atm, assuming that temperature remains constant? (Ans: 5.14 L)
- 6.2 In a vaccum chamber which is connected to a cryogenic pump, pressure as low as 1.00 nPa is being attained. Calculate the number of molecules in 1.00 m³ vessel at this pressure and temperature of 300 K. (Ans: 2.41×10^{14} molecules)
- 6.3 A gas undergoes a thermodynamic process where it absorbs 500 J of heat energy and performs 300 J work on its surroundings. Calculate the change in internal energy of the gas. (Ans: 200 J)
- 6.4 A Carnot engine is operating between a high temperature reservoir at 600 K and a low temperature reservoir at 300 K. Calculate:
- (i) the maximum possible efficiency

- (ii) the amount of work output if the engine absorbs 500 J of heat from the high temperature reservoir. [Ans: (i) 50% (ii) 250 J]
- 6.5 A refrigerator extracts 1200 J of heat from its interior (the cold reservoir) and releases 1800 J of heat to the surrounding environment (the hot reservoir) during each cycle. Calculate:
- (i) the work input required per cycle.
(ii) the coefficient of performance (COP) of the refrigerator. [Ans: (i) 600 J (ii) 2]
- 6.6 Calculate the entropy change when 1.0 mole of ice at 0°C melts to form liquid water at the same temperature. (Heat of fusion of ice per mole = 6.01×10^3 J) [Ans: 22.0 J K⁻¹]
- 6.7 A gas occupies 400 mL at 20 °C. What volume will it occupy at 80 °C, assuming constant pressure? [Ans: 482 mL]
- 6.8 A gas has a pressure of 2 atm at 300 K. What pressure will it have at 450 K, assuming constant volume? [Ans: 3 atm]

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